Chapter 3 - Parallel and Perpendicular Lines - Get Ready for Chapter 3

Refer to the figure to identify each of the following.

1. How many planes are shown in this figure?
2. Name three points that are collinear.
3. Are points C and D coplanar? Explain.
4. PHOTOGRAPHY Tina is taking a picture of her friends. If she sets a tripod level on the ground, will the bottom of each of the three legs of the tripod be coplanar?

Find each angle measure.

5. $\angle 1$
6. $\angle 2$
7. $\angle 3$
8. $\angle 4$

For each equation, find the value of $x$ for the given values of $a$ and $b$.

9. $a + 8 = -4(x - b)$, for $a = 8$ and $b = 3$
10. $b = 3x + 4a$, for $a = -9$ and $b = 12$
11. $\frac{a + 2}{b + 13} = 5x$, for $a = 18$ and $b = -1$

12. MINIATURE GOLF A miniature golf course offers a $1 ice cream cone with each round of golf purchased. If five friends each had a cone after golfing and spend a total of $30, how much does one round of golf cost?
3-1 Parallel Lines and Transversals - Check Your Understanding

Refer to the figure at the right to identify each of the following.

1. a plane parallel to plane ZWX

2. a segment skew to $\overline{TS}$ that contains point $W$

3. all segments parallel to $\overline{SV}$

4. **CONSTRUCTION** Use the diagram of the partially framed storage shed shown to identify each of the following.
   a. Name three pairs of parallel planes.
   b. Name three segments parallel to $\overline{DE}$.
   c. Name two segments parallel to $\overline{FE}$.
   d. Name two pairs of skew segments.

Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

5. $\angle 1$ and $\angle 8$

6. $\angle 2$ and $\angle 4$

7. $\angle 3$ and $\angle 6$

8. $\angle 6$ and $\angle 7$
Name:

Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles.

9. $\angle 2$ and $\angle 4$
10. $\angle 5$ and $\angle 6$
11. $\angle 4$ and $\angle 7$
12. $\angle 2$ and $\angle 7$
3-1 Parallel Lines and Transversals - Practice and Problem Solving

Refer to the figure to identify each of the following.

13. all segments parallel to $\overline{DM}$

14. a plane parallel to plane $ACD$

15. a segment skew to $\overline{BC}$

16. all planes intersecting plane $EDM$

17. all segments skew to $\overline{AE}$

18. a segment parallel to $\overline{EN}$

19. a segment parallel to $\overline{AB}$ through point $J$

20. a segment skew to $\overline{CL}$ through point $E$

Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

21. $\angle 4$ and $\angle 9$

22. $\angle 5$ and $\angle 7$

23. $\angle 3$ and $\angle 5$
Name:

26. $\angle 6$ and $\angle 8$
27. $\angle 2$ and $\angle 3$
28. $\angle 9$ and $\angle 10$
29. $\angle 4$ and $\angle 11$
30. $\angle 7$ and $\angle 11$

**SAFETY** Identify the transversal connecting each pair of angles in the photo of a fire escape shown. Then classify the relationship between each pair of angles.

31. $\angle 1$ and $\angle 2$
32. $\angle 2$ and $\angle 4$
33. $\angle 4$ and $\angle 5$
34. $\angle 6$ and $\angle 7$
35. $\angle 7$ and $\angle 8$
36. $\angle 2$ and $\angle 3$

37. **POWER** Use the photo and information at the left.
   a. What must be the relationship between power lines $p$ and $m$? Explain your reasoning.
   b. What is the relationship between line $q$ and lines $p$ and $m$?
Describe the relationship between each pair of segments as parallel, skew, or intersecting.

38. $\overline{FG}$ and $\overline{BC}$

39. $\overline{AB}$ and $\overline{CG}$

40. $\overline{DH}$ and $\overline{HG}$

41. $\overline{DF}$ and $\overline{BF}$

42. $\overline{EF}$ and $\overline{BC}$

43. $\overline{CD}$ and $\overline{AD}$

44. OPTICAL ILLUSION The illusion at the right is created using squares and straight lines.
   a. How are $\overline{AB}$ and $\overline{CD}$ related? Justify your reasoning.
   b. How are $\overline{MN}$ and $\overline{QR}$ related? $\overline{AB}$, $\overline{CD}$, and $\overline{OP}$?

45. ESCALATORS Escalators consist of steps on a continuous loop that is driven by a motor. At the top and bottom of the platform, the steps collapse to provide a level surface for entrance and exit.
   a. What is the relationship between the treads of the ascending stairs?
   b. What is the relationship between the treads of the two steps at the top of the incline?
   c. How do the treads of the steps on the incline of the escalator relate to the treads of the steps on the bottom of the conveyor?

46. OPEN ENDED Plane $P$ contains lines $a$ and $b$. Line $c$ intersects plane $P$ at point $J$. Lines $a$ and $b$ are parallel, lines $a$ and $c$ are skew, and lines $b$ and $c$ are not skew. Draw a figure based upon this description.
47. **CHALLENGE** Suppose points $A$, $B$, and $C$ lie in plane $P$, and points $D$, $E$, and $F$ lie in plane $Q$. Line $m$ contains points $D$ and $F$ and does not intersect plane $P$. Line $n$ contains points $A$ and $E$.
   a. Draw a diagram to represent the situation.
   b. What is the relationship between planes $P$ and $Q$?
   c. What is the relationship between lines $m$ and $n$?

**REASONING** Plane $X$ and plane $Y$ are parallel and plane $Z$ intersects plane $X$. Line $\overline{AB}$ is in plane $X$, line $\overline{CD}$ is in plane $Y$, and line $\overline{EF}$ is in plane $Z$. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

48. $\overline{AB}$ is skew to $\overline{CD}$.
49. $\overline{AB}$ intersects $\overline{EF}$.

50. **WRITING IN MATH** Explain why planes cannot be skew.

51. Which of the following angle pairs are alternate exterior angles?

   ![Diagram with angles labeled 1 to 12]

   A $\angle 1$ and $\angle 5$
   B $\angle 2$ and $\angle 6$
   C $\angle 2$ and $\angle 10$
   D $\angle 5$ and $\angle 9$

52. What is the measure of $\angle XYZ$?

   ![Protractor with angles 30°, 60°, 120°, 150°]
   F 30°
   G 60°
   H 120°
   J 150°

53. **SHORT RESPONSE** Name the coordinates of the points representing the $x$- and $y$-intercepts of the graph shown below.
54. SAT/ACT Of the following, the one that is not equivalent to 485 is:
   A \((3 \times 100) + (4 \times 10) + 145\)
   B \((3 \times 100) + (18 \times 10) + 5\)
   C \((4 \times 100) + (8 \times 10) + 15\)
   D \((4 \times 100) + (6 \times 10) + 25\)

55. Find the measure of each numbered angle.

   \[ m\angle 9 = 2x - 4, \]
   \[ m\angle 10 = 2x + 4 \]

56. \[ m\angle 11 = 4x, \]
    \[ m\angle 12 = 2x - 6 \]

57. \[ m\angle 19 = 100 + 20x, \]
    \[ m\angle 20 = 20x \]

58. PROOF Prove the following.
   Given: \(\overline{WY} \equiv \overline{ZX}\)
   \(A\) is the midpoint of \(\overline{WY}\).
   \(A\) is the midpoint of \(\overline{ZX}\).
   Prove: \(\overline{WA} \equiv \overline{ZA}\)

59. ALGEBRA Use the figure at the right.

59. If \(m \angle CFD = 12a + 45\), find \(a\) so that \(\overline{FC} \perp \overline{FD}\).

60. If \(m \angle AFB = 8x - 6\) and \(m \angle BFC = 14x + 8\), find the value of \(x\) so that \(\angle AFC\) is a right angle.
Find $x$.

61.

62.

63.
Explore 3-2 Geometry Software Lab: Angles and Parallel Lines - Analyze the Results

**Analyze the Results**

**ACTIVITY** Parallel Lines and a Transversal

1. Draw a line.
   Draw and label points F and G. Then use the line tool to draw FG.

2. Draw a parallel line.
   Draw a point that is not on FG and label it J. Select FG and point J, and then choose Parallel Line from the Construct menu. Draw and label a point K on this parallel line.

3. Draw a transversal.
   Draw and label point A on FG and point B on JK. Select A and B and then choose Line from the Construct menu to draw transversal AB. Then draw and label points C and D on AB as shown.

4. Measure each angle.
   Measure all eight angles formed by these lines. For example, select points F, A, then C, and choose Angle from the Measure menu to find m∠FAC.

![Diagram of angles and parallel lines with points labeled]

1. Record the measures from Step 4 in a table like this one. Which angles have the same measure?

<table>
<thead>
<tr>
<th>Angles</th>
<th>m∠FAC</th>
<th>m∠CAG</th>
<th>m∠GAB</th>
<th>m∠FAB</th>
<th>m∠JBA</th>
<th>m∠ABK</th>
<th>m∠KBD</th>
<th>m∠JBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Drag point C or F to move transversal AB so that it intersects the two parallel lines at a different angle. Add a row 2nd Measure to your table and record the new measures. Repeat these steps until your table has 3rd, 4th, and 5th Measure rows of data.

3. Using the angles listed in the table, identify and describe the relationship between all angle pairs that have the following special names. Then write a conjecture in if-then form about each angle pair when formed by any two parallel lines cut by a transversal.
   a. corresponding
   b. alternate interior
   c. alternate exterior
   d. consecutive interior

4. Drag point C or F so that the measure of any of the angles is 90.
   a. What do you notice about the measures of the other angles?
   b. Make a conjecture about a transversal that is perpendicular to one of two parallel lines.
3-2 Angles and Parallel Lines - Check Your Understanding

In the figure, $m \angle 1 = 94$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

1. $\angle 3$
2. $\angle 5$
3. $\angle 4$

In the figure, $m \angle 4 = 101$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

4. $\angle 6$
5. $\angle 7$
6. $\angle 5$

7. **ROADS** In the diagram, the guard rail is parallel to the surface of the roadway and the vertical supports are parallel to each other. Find the measures of angles 2, 3, and 4.

Find the value of the variable(s) in each figure. Explain your reasoning.

8. 

9.
10.
3-2 Angles and Parallel Lines - Practice and Problem Solving

In the figure, \( m \angle 11 = 22 \) and \( m \angle 14 = 18 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

11. \( \angle 4 \)
12. \( \angle 3 \)
13. \( \angle 12 \)
14. \( \angle 8 \)
15. \( \angle 6 \)
16. \( \angle 2 \)
17. \( \angle 10 \)
18. \( \angle 5 \)
19. \( \angle 1 \)

SOLAR ENERGY A solar dish collects energy by directing radiation from the sun to a receiver located at the focal point of the dish. Assume that the radiation rays are parallel. Determine the relationship between each pair of angles and explain your reasoning.

20. \( \angle 1 \) and \( \angle 2 \)
21. \( \angle 3 \) and \( \angle 5 \)
Find the value of the variable(s) in each figure. Explain your reasoning.

24.

\[ (x + 12)^\circ \quad 114^\circ \quad y^\circ \]

25.

\[ (3x - 15)^\circ \quad (y + 25)^\circ \quad 105^\circ \]

26.

\[ (2x)^\circ \quad 54^\circ \]

27.

\[ 96^\circ \quad (2x)^\circ \quad 94^\circ \quad (3y + 44)^\circ \]

28.

\[ (5y)^\circ \quad (2x)^\circ \quad 120^\circ \quad 108^\circ \]

29.

\[ (3y + 40)^\circ \quad (3x - 70)^\circ \quad 120^\circ \quad a^\circ \]
30. **PROOF** Copy and complete the proof of Theorem 3.2.

Given: \( m \parallel n; \ l \) is a transversal.
Prove: \( \angle 1 \) and \( \angle 2 \) are supplementary;
\( \angle 3 \) and \( \angle 4 \) are supplementary.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ?</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle 1 ) and ( \angle 3 ) form a linear pair; ( \angle 2 ) and ( \angle 4 ) form a linear pair.</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. ?</td>
<td>c. If two angles form a linear pair, then they are supplementary.</td>
</tr>
<tr>
<td>d. ( \angle 1 \equiv \angle 4, \ \angle 2 \equiv \angle 3 )</td>
<td>d. ?</td>
</tr>
<tr>
<td>e. ( m\angle 1 = m\angle 4, m\angle 2 = m\angle 3 )</td>
<td>e. Definition of Congruence</td>
</tr>
<tr>
<td>f. ?</td>
<td>f. ?</td>
</tr>
</tbody>
</table>

**STORAGE** When industrial shelving needs to be accessible from either side, additional support is provided on the side by transverse members. Determine the relationship between each pair of angles and explain your reasoning.

31. \( \angle 1 \) and \( \angle 8 \)
32. \( \angle 1 \) and \( \angle 5 \)
33. \( \angle 3 \) and \( \angle 6 \)
34. \( \angle 1 \) and \( \angle 2 \)
35. **PROOF** Write a two-column proof of the Alternate Exterior Angles Theorem.
36. **BRIDGES** Refer to the diagram of the double decker Michigan Avenue Bridge in Chicago, Illinois. The two levels of the bridge are parallel.
   a. How are the measures of the odd-numbered angles related? Explain.
   b. How are the measures of the even-numbered angles related? Explain.
   c. How are any pair of angles in which one is odd and the other is even related?
   d. What geometric term(s) can be used to relate the two roadways contained by the bridge?

![Diagram of the double decker Michigan Avenue Bridge]

37. **PROOF** In a plane, prove that if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.
   
   Find $x$. (Hint: Draw an auxiliary line.)

38. 

![Diagram with angles]

39. 

![Diagram with angles]

40. **PROBABILITY** Suppose you were to pick any two angles in the figure below.
   a. How many possible angle pairings are there? Explain.
   b. Describe the possible relationships between the measures of the angles in each pair. Explain.
   c. Describe the likelihood of randomly selecting a pair of congruent angles. Explain your reasoning.

![Diagram with angles]

41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between same-side exterior angles.
   a. **GEOMETRY** Draw five pairs of parallel lines, $m$ and $n$, $a$ and $b$, $r$ and $s$, $j$ and $k$, and $x$ and $y$, cut by a transversal $t$, and measure each angle pair.
   b. **TABULAR** Record your data in a table.
   c. **VERBAL** Make a conjecture about the relationship between the pair of angles formed on the exterior of parallel lines and on the same side of the transversal.
   d. **LOGICAL** What type of reasoning did you use to form your conjecture? Explain.
   e. **PROOF** Write a proof of your conjecture.
42. **WRITING IN MATH** If line \(a\) is parallel to line \(b\) and \(\angle 1 \cong \angle 2\), describe the relationship between lines \(b\) and \(c\). Explain your reasoning.

![Diagram of parallel lines and angles](image)

43. **WRITING IN MATH** Compare and contrast the Alternate Interior Angles Theorem and the Consecutive Interior Angles Theorem.

44. **OPEN ENDED** Draw a pair of parallel lines cut by a transversal and measure the two exterior angles on the same side of the transversal. Include the measures on your drawing. Based on the pattern you have seen for naming other pairs of angles, what do you think the name of the pair you measured would be?

45. **CHALLENGE** Find \(x\) and \(y\).

![Diagram of angles and variables](image)

46. **REASONING** Determine the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal. Explain.

47. Suppose \(\angle 4\) and \(\angle 5\) form a linear pair. If \(m \angle 1 = 2x\), \(m \angle 2 = 3x - 20\), and \(m \angle 3 = x - 4\), what is \(m \angle 3\)?

A. \(26^\circ\)
B. \(28^\circ\)
C. \(30^\circ\)
D. \(32^\circ\)

48. **SAT/ACT** A farmer raises chickens and pigs. If his animals have a total of 120 heads and a total of 300 feet, how many chickens does the farmer have?

F. 60
G. 70
H. 80
J. 90

49. **SHORT RESPONSE** If \(m \parallel n\), then which of the following statements must be true?

![Diagram of parallel lines and angles](image)

I. \(\angle 3\) and \(\angle 6\) are Alternate Interior Angles.
II. \(\angle 4\) and \(\angle 6\) are Consecutive Interior Angles.
III. \(\angle 1\) and \(\angle 7\) are Alternate Exterior Angles.
50. ALGEBRA If \(-2 + x = -6\), then \(-17 - x = \) 
   A \(-13\)  
   B \(-4\)  
   C \(13\)  
   D \(21\)

51. AVIATION Airplanes are assigned an altitude level based on the direction they are flying. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning.

Use the given information to find the measure of each numbered angle.

52. If \(\angle 1\) and \(\angle 2\) form a linear pair and \(m \angle 2 = 67\).

53. \(\angle 6\) and \(\angle 8\) are complementary; \(m \angle 8 = 47\).

54. \(m \angle 4 = 32\)

55. TRAINS A train company wants to provide routes to New York City, Dallas, Chicago, Los Angeles, San Francisco, and Washington, D.C. An engineer draws lines between each pair of cities on a map. No three of the cities are collinear. How many lines did the engineer draw?

Simplify each expression.

56. \(\frac{6 - 5}{4 - 2}\)

57. \(\frac{-5 - 2}{4 - 7}\)

58. \(\frac{-11 - 4}{12 - (-9)}\)

59. \(\frac{16 - 12}{15 - 11}\)

60. \(\frac{10 - 22}{8 - 17}\)
61. \[\frac{8 - 17}{12 - (-3)}\]

Explore 3-3 Graphing Technology Lab: Investigating Slope
- Analyze the Results

**ACTIVITY**

**Step 1** Have one group member stand at the 1-meter mark. When another group member presses the button to begin collecting data, the walker begins to walk away from the device. Walk at a slow, steady pace.

**Step 2** Stop collecting data when the walker passes the 6-meter mark. Save the data as Trial 1.

**Step 3** Repeat the experiment, walking more quickly. Save the data as Trial 2.

**Step 4** For Trial 3, repeat the experiment by walking toward the data collection device slowly.

**Step 5** Repeat the experiment, walking quickly toward the device. Save the data as Trial 4.

**Analyze the Results**

<table>
<thead>
<tr>
<th>Trial</th>
<th>Point A ((x_1, y_1))</th>
<th>Point B ((x_2, y_2))</th>
<th>Slope (\frac{y_2 - y_1}{x_2 - x_1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Compare and contrast the graphs for Trials 1 and 2. How do the graphs for Trials 1 and 3 compare?

2. Use the **TRACE** feature of the calculator to find the coordinates of two points on each graph. Record the coordinates in a table like the one shown. Then use the points to find the slope of the line.

3. Compare and contrast the slopes for Trials 1 and 2. How do the slopes for Trials 1 and 2 compare to the slopes for Trials 3 and 4?

4. The slope of a line describes the rate of change of the quantities represented by the \(x\)- and \(y\)-values. What is represented by the rate of change in this experiment?

5. **MAKE A CONJECTURE** What would the graph look like if you were to collect data while the walker was standing still? Use the data collection device to test your conjecture.
Find the slope of each line.

1.
4. **BOTANY** Kudzu is a fast-growing vine found in the southeastern United States. An initial measurement of the length of a kudzu vine was 0.5 meter. Seven days later the plant was 4 meters long.
   a. Graph the line that models the length of the plant over time.
   b. What is the slope of your graph? What does it represent?
   c. Assuming that the growth rate of the plant continues, how long will the plant be after 15 days?

Determine whether \( WX \) and \( YZ \) are parallel, perpendicular, or neither. Graph each line to verify your answer.

5. \( W(2, 4), X(4, 5), Y(4, 1), Z(8, -7) \)

6. \( W(1, 3), X(-2, -5), Y(-6, -2), Z(8, 3) \)

7. \( W(-7, 6), X(-6, 9), Y(6, 3), Z(3, -6) \)

8. \( W(1, -3), X(0, 2), Y(-2, 0), Z(8, 2) \)

Graph the line that satisfies each condition.

9. passes through \( A(3, -4) \), parallel to \( BC \) with \( B(2, 4) \) and \( C(5, 6) \)

10. slope = 3, passes through \( A(-1, 4) \)

11. passes through \( P(7, 3) \), perpendicular to \( LM \) with \( L(-2, -3) \) and \( M(-1, 5) \)
3-3 Slopes of Lines - Practice and Problem Solving

Find the slope of each line.

12.

13.

14.

15.

16.
17. Determine the slope of the line that contains the given points.

18. \( C(3, 1), D(-2, 1) \)
19. \( E(5, -1), F(2, -4) \)
20. \( G(-4, 3), H(-4, 7) \)
21. \( J(7, -3), K(-8, -3) \)
22. \( L(8, -3), M(-4, -12) \)
23. \( P(-3, -5), Q(-3, -1) \)
24. \( R(2, -6), S(-6, 5) \)
25. \( T(-6, -11), V(-12, -10) \)

26. **MOUNTAIN BIKING** In 2004, 8 million Americans over the age of 7 participated in mountain biking, and in 2006, 8.5 million participated.
   a. Create a graph to show the number of participants in mountain biking based on the change in participation from 2004 to 2006.
   b. Based on the data, what is the growth per year of the sport?
   c. If participation continues at the same rate, what will be the participation in 2013 to the nearest 10,000?

   a. Graph a trend line to predict the price of the MP3 player for 2003 through 2007.
   b. Based on the data, how much does the price drop per year?
   c. If the trend continues, what will be the cost of an MP3 player in 2010?

   Determine whether \( \overline{AB} \) and \( \overline{CD} \) are parallel, perpendicular, or neither. Graph each line to verify your answer.

28. \( A(1, 5), B(4, 4), C(9, -10), D(-6, -5) \)
29. \( A(-6, -9), B(8, 19), C(0, -4), D(2, 0) \)
30. \( A(4, 2), B(-3, 1), C(6, 0), D(-10, 8) \)
31. \( A(8, -2), B(4, -1), C(3, 11), D(-2, -9) \)
32. \(A(8, 4), B(4, 3), C(4, -9), D(2, -1)\)

33. \(A(4, -2), B(-2, -8), C(4, 6), D(8, 5)\)

**Graph the line that satisfies each condition.**

34. passes through \(A(2, -5)\), parallel to \(\overline{BC}\) with \(B(1, 3)\) and \(C(4, 5)\)

35. slope = \(-2\), passes through \(H(-2, -4)\)

36. passes through \(K(3, 7)\), perpendicular to \(\overline{LM}\) with \(L(-1, -2)\) and \(M(-4, 8)\)

37. passes through \(X(1, -4)\), parallel to \(\overline{YZ}\) with \(Y(5, 2)\) and \(Z(-3, -5)\)

38. slope = \(\frac{2}{3}\), passes through \(J(-5, 4)\)

39. passes through \(D(-5, -6)\), perpendicular to \(\overline{FG}\) with \(F(-2, -9)\) and \(G(1, -5)\)

40. **STADIUMS** Before it was demolished, the RCA Dome was home to the Indianapolis Colts. The attendance in 2001 was 450,746, and the attendance in 2005 was 457,373.

   a. What is the approximate rate of change in attendance from 2001 to 2005?
   b. If this rate of change continues, predict the attendance for 2012.
   c. Will the attendance continue to increase indefinitely? Explain.
   d. The Colts have now built a new, larger stadium. Do you think their decision was reasonable? Why or why not?

**Determine which line passing through the given points has a steeper slope.**

41. Line 1: \((0, 5)\) and \((6, 1)\)
   Line 2: \((-4, 10)\) and \((8, -5)\)

42. Line 1: \((0, -4)\) and \((2, 2)\)
   Line 2: \((0, -4)\) and \((4, 5)\)

43. Line 1: \((-6, 7)\) and \((9, -3)\)
   Line 2: \((-9, 9)\) and \((3, 5)\)

44. Line 1: \((-9, -4)\) and \((7, 0)\)
   Line 2: \((0, 1)\) and \((7, 4)\)
45. **ENDANGERED SPECIES** Michigan provides habitat for two endangered species, the bald eagle and the gray wolf. The graph shows the Michigan population of each species in 1992 and 2006.
   a. Which species experienced a greater rate of change in population?
   b. Make a line graph showing the growth of both populations.
   c. If both species continue to grow at their respective rates, what will the population of each species be in 2012?

![Graph showing population of Michigan Bald Eagles and Gray Wolves from 1992 to 2006.]

Find the value of \(x\) or \(y\) that satisfies the given conditions. Then graph the line.

46. The line containing \((4, -1)\) and \((x, -6)\) has a slope of \(\frac{-5}{2}\).

47. The line containing \((-4, 9)\) and \((4, 3)\) is parallel to the line containing \((-8, 1)\) and \((4, y)\).

48. The line containing \((8, 7)\) and \((7, -6)\) is perpendicular to the line containing \((2, 4)\) and \((x, 3)\).

49. The line containing \((1, -3)\) and \((3, y)\) is parallel to the line containing \((5, -6)\) and \((9, y)\).

50. **SCHOOLS** In 2000, Jefferson High School had 1125 students. By 2006, the student body had increased to 1425 students. When Fairview High School was built in 2001, it had 1275 students. How many students did Fairview High School have in 2006 if the student body grew at the same rate as Jefferson High School?

51. **MUSIC** Maggie and Mikayla want to go to the music store near Maggie’s house after school. They can walk 3.5 miles per hour and ride their bikes 10 miles per hour.
   a. Create a table to show how far Maggie and Mikayla can travel walking and riding their bikes. Include distances for 0, 1, 2, 3, and 4 hours.
   b. Create a graph to show how far Maggie and Mikayla can travel based on time for both walking and riding their bikes. Be sure to label the axes of your graph.
   c. What does the slope represent in your graph?
   d. Maggie’s mom says they can only go if they can make it to the music store and back in less than two hours. If they want to spend at least 30 minutes in the music store and it is four miles away, can they make it? Should they walk or ride their bikes? Explain your reasoning.

52. **WRITE A QUESTION** A classmate says that all lines have positive or negative slope. Write a question that would challenge his conjecture.
53. **FIND THE ERROR** Terrell and Hale calculated the slope of the line passing through the points \( Q(3, 5) \) and \( R(-2, 2) \). Is either of them correct? Explain your reasoning.

Terrell:
\[
\begin{align*}
\frac{y_2 - y_1}{x_2 - x_1} &= \frac{5 - 5}{3 - (-2)} \\
&= \frac{0}{5} \\
&= 0
\end{align*}
\]

Hale:
\[
\begin{align*}
\frac{y_2 - y_1}{x_2 - x_1} &= \frac{5 - 2}{3 - (-2)} \\
&= \frac{3}{5}
\end{align*}
\]

54. **REASONING** Draw a square \( ABCD \) with vertices at \( A(2, -4) \) and \( C(10, 4) \).
   a. Find the other two vertices of the square and label them \( B \) and \( D \).
   b. Show that \( AD \parallel BC \) and \( AB \parallel DC \).
   c. Show that the measure of each angle inside the square is equal to \( 90^\circ \).

55. **WRITING IN MATH** Describe the slopes of the Sears Tower and the Leaning Tower of Pisa.

56. **CHALLENGE** In this lesson you learned that \( m = \frac{y_2 - y_1}{x_2 - x_1} \). Use an algebraic proof to show that the slope can also be calculated using the equation \( m = \frac{y_1 - y_2}{x_1 - x_2} \).

57. **WRITING IN MATH** Find two additional points that lie along the same line as \( X(3, -1) \) and \( Y(-1, 7) \). Generalize a method you can use to find additional points on the line from any given point.

58. The graph of which equation passes through \((-3, -2)\) and is perpendicular to the graph of \( y = \frac{3}{4}x + 8 \)?

A. \( y = -\frac{4}{3}x - 6 \)
B. \( y = \frac{4}{3}x + 5 \)
C. \( y = \frac{3}{4}x + \frac{1}{4} \)
D. \( y = -\frac{3}{4}x - 5 \)

59. **SHORT RESPONSE** A set of 25 cards is randomly placed face down on a table. 15 cards have only the letter A written on the face, and 10 cards have only the letter B. Patrick turned over 1 card. What are the odds of this card having the letter B written on its face?
60. **ALGEBRA** Jamie is collecting money to buy an $81 gift for her teacher. She has already contributed $24. She will collect $3 from each contributing student. From how many students must Jamie collect money?
   F) 3 students
   G) 9 students
   H) 12 students
   J) 19 students

61. **SAT/ACT** The area of a circle is $20\pi$ square centimeters. What is its circumference?
   \[ A = 20\pi \text{ cm}^2 \]
   A) $\sqrt{5}\pi$ cm
   B) $2\sqrt{5}\pi$ cm
   C) $4\sqrt{5}\pi$ cm
   D) 20$\pi$ cm

   In the figure, $a \parallel b$, $c \parallel d$, and $m\angle 4 = 57$. Find the measure of each angle.

62. $\angle 5$
63. $\angle 1$
64. $\angle 8$
65. $\angle 10$

66. Refer to the diagram at the right.

   Name all segments parallel to $\overline{TU}$.

67. Name all planes intersecting plane $BCR$.

68. Name all segments skew to $\overline{DE}$.
Name:

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

69. Given: \( \angle B \) and \( \angle C \) are vertical angles.
   Conclusion: \( \angle B \equiv \angle C \)

70. Given: \( \angle W \equiv \angle Y \)
   Conclusion: \( \angle W \) and \( \angle Y \) are vertical angles.

71. CONSTRUCTION There are four buildings on the Mansfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building?

   Solve for \( y \).

72. \( 3x + y = 5 \)

73. \( 4x + 2y = 6 \)

74. \( 4y - 3x = 5 \)
Chapter 3 - Parallel and Perpendicular Lines - Mid-Chapter
Quiz: Lessons 3-1 through 3-3

Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

1. \( \angle 6 \) and \( \angle 3 \)
2. \( \angle 1 \) and \( \angle 14 \)
3. \( \angle 10 \) and \( \angle 11 \)
4. \( \angle 5 \) and \( \angle 7 \)

Refer to the figure to identify each of the following.

5. a plane parallel to plane \( ABCD \)
6. a segment skew to \( \overline{GH} \) that contains point \( D \)
7. all segments parallel to \( \overline{HE} \)

8. **MULTIPLE CHOICE** Which term best describes \( \angle 4 \) and \( \angle 8 \)?

   A corresponding 
   B alternate exterior 
   C alternate interior 
   D consecutive interior

In the figure, \( m \angle 4 = 104 \), \( m \angle 14 = 118 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.
10. \( \angle 9 \)

11. \( \angle 10 \)

12. \( \angle 7 \)

Find \( x \).

13. [Diagram: Two angles labeled \((3x - 9)^\circ\) and \((2.5x + 15)^\circ\).]

14. **MODEL TRAINS** Amy is going to set up two parallel train tracks with a third track running diagonally across the first two. To properly place a switch, she needs the angle between the diagonal and the top of the second track to be twice as large as the angle between the diagonal and bottom of the first track. What is the value of \( x \)?

Determine whether \( \overrightarrow{AB} \) and \( \overrightarrow{XY} \) are parallel, perpendicular, or neither. Graph each line to verify your answer.

15. \( A(2, 0), B(4, -5), X(-3, 3), Y(-5, 8) \)

16. \( A(1, 1), B(6, -9), X(4, -10), Y(7, -4) \)

Find the slope of each line.

17. line \( \ell \)

18. a line parallel to \( m \)

19. a line perpendicular to \( n \)

20. **SALES** The 2003 and 2006 sales figures for Vaughn Electronics are in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Approximate Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>240,000</td>
</tr>
<tr>
<td>2006</td>
<td>330,000</td>
</tr>
</tbody>
</table>

a. What is the rate of change in approximate sales from 2003 to 2006?
b. If this rate of change continues, predict the approximate sales for the year 2010.
3-4 Equations of Lines - Check Your Understanding

Write an equation in slope-intercept form of the line having the given slope and y-intercept. Then graph the line.

1. \( m: 4 \), \( y\)-intercept: \(-3\)
2. \( m: \frac{1}{2} \), \( y\)-intercept: \(-1\)
3. \( m: \frac{2}{3} \), \( y\)-intercept: \(5\)

Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

4. \( m = 5 \), \((3, -2)\)
5. \( m = \frac{1}{4} \), \((-2, -3)\)
6. \( m = -4.25 \), \((-4, 6)\)

Write an equation of the line through each pair of points in slope-intercept form.

7. \((0, -1)\) and \((4, 4)\)
8. \((4, 3)\) and \((1, -6)\)
9. \((6, 5)\) and \((-1, -4)\)

10. Write an equation in slope-intercept form for a line containing \((3, 2)\) that is perpendicular to the line with equation \(y = -2x + 6\).

11. Write an equation in slope-intercept form for a line containing \((-1, 5)\) that is parallel to the line with equation \(y = 4x - 5\).

12. MUSIC Kameko currently subscribes to Ace Music, an online music service, but she is considering switching to another online service, Orange Tunes. The plan for each online music service is described below.

\[\begin{align*}
\text{Ace Music} & : \text{Subscription fee $6.95/mo, $0.79 per song downloaded} \\
\text{Orange Tunes} & : \text{Subscription fee $10.95/mo, 40 downloads per month}
\end{align*}\]

a. Write an equation to represent the total monthly cost for each plan.
b. Graph the equations.
c. If Kameko downloads 15 songs per month, should she keep her current plan, or change to the other plan?
3-4 Equations of Lines - Practice and Problem Solving

Write an equation in slope-intercept form of the line having the given slope and \( y \)-intercept or given points. Then graph the line.

13. \( m = -5 \), \( y \)-intercept: \(-2\)
14. \( m = -7 \), \( b = -4 \)
15. \( m = 9 \), \( b = 2 \)
16. \( m = 12 \), \( y \)-intercept: \( \frac{4}{5} \)
17. \( m = \frac{3}{4} \), \((0, 4)\)
18. \( m = \frac{5}{11} \), \((0, -3)\)

Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

19. \( m = 2 \), \((3, 11)\)
20. \( m = 4 \), \((-4, 8)\)
21. \( m = -7 \), \((1, 9)\)
22. \( m = \frac{5}{7} \), \((-2, -5)\)
23. \( m = -\frac{4}{5} \), \((-3, -6)\)
24. \( m = -2.4 \), \((14, -12)\)

Write an equation of the line through each pair of points in slope-intercept form.

25. \((-1, -4)\) and \((3, -4)\)
26. \((2, -1)\) and \((2, 6)\)
27. \((-3, -2)\) and \((-3, 4)\)
28. \((0, 5)\) and \((3, 3)\)
29. \((-12, -6)\) and \((8, 9)\)
Write an equation in slope-intercept form for each line shown or described.

31. \( \overrightarrow{EF} \)

32. \( \overrightarrow{MN} \)

33. contains \((-1, -2)\) and \((3, 4)\)

34. contains \((-4, -5)\) and \((-8, -13)\)

35. \(x\)-intercept = 3, \(y\)-intercept = -2

36. \(x\)-intercept = \(-\frac{1}{2}\), \(y\)-intercept = 4

Write an equation in slope-intercept form for each line described.

37. passes through \((-7, -4)\), perpendicular to \(y = \frac{1}{2}x + 9\)

38. passes through \((-1, -10)\), parallel to \(y = 7\)

39. passes through \((6, 2)\), parallel to \(y = -\frac{2}{3}x + 1\)

40. passes through \((-2, 2)\), perpendicular to \(y = -5x - 8\)

41. **PLANNING** Karen is planning a graduation party for the senior class. She plans to rent a meeting room at the convention center that costs $400. There is an additional fee of $5.50 for each person who attends the party.
   a. Write an equation to represent the cost \(y\) of the party if \(x\) people attend.
   b. Graph the equation.
   c. There are 285 people in Karen’s class. If \(\frac{2}{3}\) of these people attend, how much will the party cost?
   d. If the senior class has raised $2000 for the party, how many people can attend?
42. **SAVINGS** Victor is saving his money to buy a new satellite radio for his car. He wants to save enough money for the radio and one year of satellite radio service before he buys it. He started saving for the radio with $50 that he got for his birthday. Since then, he has been adding $15 every week after he cashes his paycheck.

   a. Write an equation to represent Victor’s savings $y$ after $x$ weeks.
   b. Graph the equation.
   c. How long will it take Victor to save $150? 
   d. A satellite radio costs $180. Satellite radio service costs $10 per month. If Victor started saving two weeks ago, how much longer will it take him to save enough money? Explain.

Name the line(s) on the graph shown that match each description.

43. parallel to $y = 2x - 3$

44. perpendicular to $y = \frac{1}{2}x + 7$

45. intersecting but not perpendicular to $y = \frac{1}{2}x - 5$

**Determine whether the lines are parallel, perpendicular, or neither.**

46. $y = 2x + 4, y = 2x - 10$

47. $y = -\frac{1}{2}x - 12, y = 2x + 7$

48. $y - 4 = 3(x + 5), y + 3 = -\frac{1}{3}(x + 1)$

49. $y - 3 = 6(x + 2), y + 3 = -\frac{1}{3}(x - 4)$

50. Write an equation in slope-intercept form for a line containing (4, 2) that is parallel to the line $y - 2 = 3(x + 7)$.

51. Write an equation for a line containing (−8, 12) that is perpendicular to the line containing the points (3, 2) and (−7, 2).

52. Write an equation in slope-intercept form for a line containing (5, 3) that is parallel to the line $y + 11 = \frac{1}{2}(4x + 6)$. 
53. **POTTERY** A local community arts center offers pottery classes. A $40 enrollment fee covers supplies and materials, including one ten-pound bag of clay. Any extra bags of clay cost $15 each. Write an equation to represent the cost of the class and $x$ bags of clay.

54. **MULTIPLE REPRESENTATIONS** Ms. Smith has asked Dan to work on Friday night valet parking cars for a birthday party. Ms. Smith has given Dan two payment options: he can choose to be paid $3 per car or a $75 fee plus $1.50 per car.
   a. **TABULAR** Create tables to show how much Dan would be paid for 20, 50, and 100 cars for both options.
   b. **NUMERICAL** Write an equation to represent Dan’s earnings for each payment option.
   c. **GRAPHICAL** Graph the equations for both options.
   d. **ANALYTICAL** If 35 people attend the party, which option is more profitable? If 75 people attend? Explain.
   e. **VERBAL** Write a statement to describe which payment option is more profitable based on the number of cars that Dan parks.
   f. **LOGICAL** If Ms. Smith sent out 50 invitations, which option should Dan choose? Explain your reasoning.

55. **CHALLENGE** Find the value of $n$ so that the line perpendicular to the line with the equation $-2y + 4 = 6x + 8$ passes through the points at $(n, -4)$ and $(2, -8)$.

56. **REASONING** Determine whether the points at $(-2, 2)$, $(2, 5)$, and $(6, 8)$ are collinear. Justify your answer.

57. **OPEN ENDED** Write equations for two different pairs of perpendicular lines that intersect at the point at $(-3, -7)$.

58. **FIND THE ERROR** Mark and Josefina wrote an equation of a line with slope $-5$ that passes through the point at $(-2, 4)$. Is either of them correct? Explain your reasoning.

59. **WRITING IN MATH** When is it easier to use the point-slope form to write an equation of a line and when is it easier to use the slope-intercept form?
60. Which graph best represents a line passing through the point \((-2, -3)\)?

A

B

C

D

61. Which equation describes the line that passes through the point at \((-2, 1)\) and is perpendicular to the line \(y = \frac{1}{3}x + 5\)?

F \(y = 3x + 7\)
G \(y = \frac{1}{3}x + 7\)
H \(y = -3x - 5\)
J \(y = -\frac{1}{3}x - 5\)

62. **GRIDDED RESPONSE** At Jefferson College, 80% of students have cell phones. Of the students who have cell phones, 70% have computers. What percent of the students at Jefferson College have both a cell phone and a computer?
Name:

63. SAT/ACT Which expression is equivalent to \(4(x - 6) - \frac{1}{2}(x^2 + 8)\)?
   A \(4x^2 + 4x - 28\)
   B \(-\frac{1}{2}x^2 + 4x - 28\)
   C \(-\frac{1}{2}x^2 + 6x - 24\)
   D \(3x - 20\)

Determine the slope of the line that contains the given points.

64. \(J(4, 3), K(5, -2)\)

65. \(X(0, 2), Y(-3, -4)\)

66. \(A(2, 5), B(5, 1)\)

Find \(x\) and \(y\) in each figure.

67.

68.

69. DRIVING Lacy’s home is located at the midpoint between Newman’s Gas Station and Gas-O-Rama. Newman’s Gas Station is a quarter mile away from Lacy’s home. How far away is Gas-O-Rama from Lacy’s home? How far apart are the two gas stations?

Determine the relationship between each pair of angles.

70. \(\angle 1\) and \(\angle 12\)

71. \(\angle 7\) and \(\angle 10\)

72. \(\angle 4\) and \(\angle 8\)

73. \(\angle 2\) and \(\angle 11\)
Extend 3-4 Geometry Lab: Equations of Perpendicular Bisectors
- Exercises

Find the equation of the perpendicular bisector $\overline{PQ}$ for the given endpoints.

1. $P(5, 2), Q(7, 4)$
2. $P(-3, 9), Q(-1, 5)$
3. $P(-6, -1), Q(8, 7)$
4. $P(-2, 1), Q(0, -3)$
5. $P(0, 1.6), Q(0.5, 2.1)$
6. $P(-7, 3), Q(5, 3)$
7. Extend what you have learned to find the equations of the lines that contain the sides of $\triangle XYZ$ with vertices $X(-2, 0), Y(1, 3)$, and $Z(3, -1)$. 
3-5 Proving Lines Parallel - Check Your Understanding

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. \( \angle 1 \cong \angle 3 \)

2. \( \angle 2 \cong \angle 5 \)

3. \( \angle 3 \cong \angle 10 \)

4. \( m \angle 6 + m \angle 8 = 180 \)

5. SHORT RESPONSE Find \( x \) so that \( m \parallel n \). Show your work.
6. **PROOF** Copy and complete the proof of Theorem 3.5.

Given: \( \angle 1 \equiv \angle 2 \)

Prove: \( \ell \parallel m \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \angle 1 \equiv \angle 2 )</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle 2 \equiv \angle 3 )</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. ( \angle 1 \equiv \angle 3 )</td>
<td>c. Transitive Property</td>
</tr>
<tr>
<td>d. ?</td>
<td>d. ?</td>
</tr>
</tbody>
</table>

7. **RECREATION** Is it possible to prove that the backrest and footrest of the lounging beach chair are parallel? If so, explain how. If not, explain why not.

![Image of a beach chair with angles labeled 135°]
3-5 Proving Lines Parallel - Practice and Problem Solving

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

8. \( \angle 1 \cong \angle 2 \)

9. \( \angle 2 \cong \angle 9 \)

10. \( \angle 5 \cong \angle 7 \)

11. \( m\angle 7 + m\angle 8 = 180 \)

12. \( m\angle 3 + m\angle 6 = 180 \)
Name:

13. \( \angle 3 \cong \angle 5 \)

14. \( \angle 3 \cong \angle 7 \)

15. \( \angle 4 \cong \angle 5 \)

Find \( x \) so that \( m \parallel n \). Identify the postulate or theorem you used.

16. 

17. 

18. 

19. 

20. 

21.
22. **FRAMING** Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

23. **PROOF** Copy and complete the proof of Theorem 3.6.

Given: ∠1 and ∠2 are supplementary.
Prove: ℓ∥m

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ?</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ∠2 and ∠3 form a linear pair.</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. ?</td>
<td>c. ?</td>
</tr>
<tr>
<td>d. ∠1 ≡ ∠3</td>
<td>d. ?</td>
</tr>
<tr>
<td>e. ℓ∥m</td>
<td>e. ?</td>
</tr>
</tbody>
</table>

24. **CRAFTS** Jacqui is making a stained glass piece. She cuts the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Jacqui knows that each pair of opposite sides are parallel.

**PROOF** Write a two-column proof for each of the following.

25. **Given:** ∠1 ≡ ∠3  
    
    AC∥BD
    
    **Prove:** AB∥CD

26. **Given:** WX∥YZ  
    ∠2 ≡ ∠3  
    
    **Prove:** WY∥XZ
27. Given: \( \angle ABC \cong \angle ADC \)
\[ m\angle A + m\angle ABC = 180 \]
Prove: \( AB \parallel CD \)

28. Given: \( \angle 1 \cong \angle 2 \)
\( LJ \perp ML \)
Prove: \( KM \perp ML \)

29. MAILBOXES Mail slots are used to make the organization and distribution of mail easier. In the mail slots shown, each slot is perpendicular to each of the sides. Explain why you can conclude that the slots are parallel.

30. PROOF Write a paragraph proof of Theorem 3.8.

31. PROOF Write a two-column proof of Theorem 3.7.

32. STAIRS Based upon the information given in the photo of the staircase at the right, what is the relationship between each step? Explain your answer.

Determine whether lines \( r \) and \( s \) are parallel. Justify your answer.
36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the shortest distance between two parallel lines.
   a. **GEOMETRIC** Draw three sets of parallel lines \( K \) and \( \ell \), \( s \) and \( t \), and \( X \) and \( Y \). Draw the shortest segment \( BC \) and label points \( A \) and \( D \) as shown below.

   b. **TABULAR** Copy the table below, measure \( \angle ABC \) and \( \angle BCD \), and complete the table.

<table>
<thead>
<tr>
<th>Set of Parallel Lines</th>
<th>( m \angle ABC )</th>
<th>( m \angle BCD )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K ) and ( \ell )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s ) and ( t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( X ) and ( Y )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. **VERBAL** Make a conjecture about the angle the shortest segment forms with both parallel lines.

37. **FIND THE ERROR** Sumi and Daniela are determining which lines are parallel in the figure at the right. Sumi says that since \( \angle 1 \equiv \angle 2 \), \( WY \parallel XZ \). Daniela disagrees and says that since \( \angle 1 \equiv \angle 2 \), \( WX \parallel YZ \). Is either of them correct? Explain.

38. **REASONING** Given the statement, *In a plane, if two lines are perpendicular to the same line, then they are parallel* (Theorem 3.8), is the statement still true if the lines are not coplanar? Draw a figure to justify your answer.
39. OPEN ENDED Draw a triangle \( ABC \).
   a. Construct the line parallel to \( BC \) through point \( A \).
   b. Use measurement to justify that the line you constructed is parallel to \( BC \).
   c. Use mathematics to justify this construction.

40. REASONING Determine whether parallel lines are reflexive, symmetric, or transitive. Do parallel lines form an equivalence relation? Explain.

41. CHALLENGE Refer to the figure at the right.
   a. If \( m \angle 1 + m \angle 2 = 180 \), prove that \( a \parallel c \).
   b. Given that \( a \parallel c \), if \( m \angle 1 + m \angle 3 = 180 \), prove that \( t \perp c \).

42. WRITING IN MATH Summarize the five methods used in this lesson to prove that two lines are parallel.

43. WRITING IN MATH Describe a situation in which two parallel lines are cut by a transversal and a pair of consecutive interior angles are both supplementary and congruent. Explain.

44. Which of the following facts would be sufficient to prove that line \( d \) is parallel to \( XZ \)?

   A \( \angle 1 \equiv \angle 3 \)
   B \( \angle 3 \equiv \angle Z \)
   C \( \angle 1 \equiv \angle Z \)
   D \( \angle 2 \equiv \angle X \)

45. ALGEBRA The expression \( \sqrt{52} + \sqrt{117} \) is equivalent to
   F 13
   G \( 5\sqrt{13} \)
   H \( 6\sqrt{13} \)
   J \( 13\sqrt{13} \)

46. What is the approximate surface area of the figure?

   A 101.3 in\(^2\)
   B 108 in\(^2\)
   C 202.5 in\(^2\)
   D 216 in\(^2\)
47. SAT/ACT If \( x^2 = 25 \) and \( y^2 = 9 \), what is the greatest possible value of \( (x - y)^2 \)?  
   F 4  
   G 58  
   H 64  
   J 70

Write an equation in slope-intercept form of the line having the given slope and \( y \)-intercept.

48. \( m: 2.5, (0, 0.5) \)

49. \( m: \frac{4}{5}, (0, -9) \)

50. \( m: \frac{7}{8}, \left(0, \frac{5}{6}\right) \)

51. ROAD TRIP Anne is driving 400 miles to visit Niagara Falls. She manages to travel the first 100 miles of her trip in two hours. If she continues at this rate, how long will it take her to drive the remaining distance?

Find a counterexample to show that each conjecture is false.

52. **Given:** \( \angle 1 \) and \( \angle 2 \) are complementary angles.  
**Conjecture:** \( \angle 1 \) and \( \angle 2 \) form a right angle.

53. **Given:** points \( W, X, Y, \) and \( Z \)  
**Conjecture:** \( W, X, Y, \) and \( Z \) are noncollinear.

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

54.

55.

56.

57. Find \( x \) and \( y \) so that \( \overline{BE} \) and \( \overline{AD} \) are perpendicular.
3-6 Perpendiculars and Distance - Check Your Understanding

Copy each figure. Construct the segment that represents the distance indicated.

1. $Y$ to $\overline{TS}$

2. $C$ to $\overline{AB}$

3. MARCHING BAND After forming a line, every even member of a marching band turns to face the home team’s end zone and marches 5 paces straight forward. At the same time, every odd member turns in the opposite direction and marches 5 paces straight forward. Assuming that each band member covers the same distance, what formation should result? Justify your answer.

COORDINATE GEOMETRY Find the distance from $P$ to $\ell$.

4. Line $\ell$ contains points (4, 3) and (−2, 0). Point $P$ has coordinates (3, 10).

5. Line $\ell$ contains points (−6, 1) and (9, −4). Point $P$ has coordinates (4, 1).

6. Line $\ell$ contains points (4, 18) and (−2, 9). Point $P$ has coordinates (−9, 5).

Find the distance between each pair of parallel lines with the given equations.

7. $y = -2x + 4$
   $y = -2x + 14$

8. $y = 7$
   $y = -3$
3-6 Perpendiculars and Distance - Practice and Problem Solving

Copy each figure. Construct the segment that represents the distance indicated.
Name:

9. \( Q \) to \( \overline{RS} \)

10. \( A \) to \( \overline{BC} \)

11. \( H \) to \( \overline{FG} \)

12. \( K \) to \( \overline{LM} \)

13. **DRIVEWAYS** In the diagram at the right, is the driveway shown the shortest possible one from the house to the road? Explain why or why not.

14. **SCHOOL** Rondell is crossing the courtyard in front of his school. Three possible paths are shown in the diagram at the right. Which of the three paths shown is the shortest? Explain your reasoning.

**COORDINATE GEOMETRY** Find the distance from \( P \) to \( \ell \).

15. Line \( \ell \) contains points \((0, -3)\) and \((7, 4)\). Point \( P \) has coordinates \((4, 3)\).
Name:

16. Line $\ell$ contains points $(11, -1)$ and $(-3, -11)$. Point $P$ has coordinates $(-1, 1)$.

17. Line $\ell$ contains points $(-2, 1)$ and $(4, 1)$. Point $P$ has coordinates $(5, 7)$.

18. Line $\ell$ contains points $(4, -1)$ and $(4, 9)$. Point $P$ has coordinates $(1, 6)$.

19. Line $\ell$ contains points $(1, 5)$ and $(4, -4)$. Point $P$ has coordinates $(-1, 1)$.

20. Line $\ell$ contains points $(-8, 1)$ and $(3, 1)$. Point $P$ has coordinates $(-2, 4)$.

Find the distance between each pair of parallel lines with the given equations.

21. $y = -2$
    $y = 4$

22. $x = 3$
    $x = 7$

23. $y = 5x - 22$
    $y = 5x + 4$

24. $y = \frac{1}{3}x - 3$
    $y = \frac{1}{3}x + 2$

25. $x = 8.5$
    $x = -12.5$

26. $y = 15$
    $y = -4$

27. $y = \frac{1}{4}x + 2$
    $4y - x = -60$

28. $3x + y = 3$
    $y + 17 = -3x$

29. $y = -\frac{5}{4}x + 3.5$
    $4y + 10.6 = -5x$

30. PROOF Write a two-column proof of Theorem 3.9.

Find the distance from the line to the given point.
31. \( y = -3, \ (5, \ 2) \)

32. \( y = \frac{1}{6}x + 6, \ (-6, \ 5) \)

33. \( x = 4, \ (-2, \ 5) \)

34. **POSTERS** Alma is hanging two posters on the wall in her room as shown. How can Alma use perpendicular distances to confirm that the posters are parallel?

35. **SCHOOL SPIRIT** Brock is decorating a hallway bulletin board to display pictures of students demonstrating school spirit. He cuts off one length of border to match the board’s width, and then uses that strip as a template to cut a second strip that is exactly the same length for the bottom.

When stapling the bottom border in place, he notices that the strip he cut is about a quarter of an inch too short. Describe what he can conclude about the bulletin board. Explain your reasoning.
CONSTRUCTION Line $\ell$ contains points at $(-4, 3)$ and $(2, -3)$. Point $P$ at $(-2, 1)$ is on line $\ell$. Complete the following construction.

Step 1
Graph line $\ell$ and point $P$, and put the compass at point $P$. Using the same compass setting, draw arcs to the left and right of $P$. Label these points $A$ and $B$.

Step 2
Open the compass to a setting greater than $AP$. Put the compass at point $A$ and draw an arc above line $\ell$.

Step 3
Using the same compass setting, put the compass at point $B$ and draw an arc above line $\ell$. Label the point of intersection $Q$. Then draw $PQ$.

36. What is the relationship between line $\ell$ and $PQ$? Verify your conjecture using the slopes of the two lines.

37. Repeat the construction above using a different line and point on that line.

38. **COORDINATE GEOMETRY** $\overline{AB}$ has a slope of 2 and midpoint $M(3, 2)$. A segment perpendicular to $\overline{AB}$ has midpoint $P(4, -1)$ and shares endpoint $B$ with $\overline{AB}$.
   a. Graph the segments.
   b. Find the coordinates of $A$ and $B$. 
39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the areas of triangles formed by points on parallel lines.
   a. **GEOMETRIC** Draw two parallel lines and label them as shown.

   ![Parallel Lines Diagram]

   b. **VERBAL** Where would you place point C on line m to ensure that triangle ABC would have the largest area? Explain your reasoning.
   c. **ANALYTICAL** If \( AB = 11 \) inches, what is the maximum area of \( \triangle ABC \)?

40. **PERPENDICULARITY AND PLANES** Make a copy of the diagram below to answer each question, marking the diagram with the given information.

   ![Perpendicular Planes Diagram]

   a. If two lines are perpendicular to the same plane, then they are coplanar. If both line a and line b are perpendicular to plane \( P \), what must also be true?
   b. If a plane intersects two parallel planes, then the intersections form two parallel lines. If planes \( R \) and \( Q \) are parallel and they intersect plane \( P \), what must also be true?
   c. If two planes are perpendicular to the same line, then they are parallel. If both plane \( Q \) and plane \( R \) are perpendicular to line \( \ell \), what must also be true?

41. **FIND THE ERROR** Han draws the segments \( \overline{AB} \) and \( \overline{CD} \) shown below using a straightedge. He claims that these two lines, if extended, will never intersect. Shenequa claims that they will. Is either of them correct? Justify your answer.

   ![Lines and Segments Diagram]

42. **WRITING IN MATH** Describe a method that could be used to find the distance between a point and a plane.

43. **CHALLENGE** Suppose a line perpendicular to a pair of parallel lines intersects the lines at the points \((a, 4)\) and \((0, 6)\). If the distance between the parallel lines is \( \sqrt{5} \), find the value of \( a \) and the equations of the parallel lines.

44. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain. 
   The distance between a line and a plane can be found.

45. **OPEN ENDED** Draw an irregular convex polygon using a straightedge.
   a. Construct the distance between one vertex and a nonadjacent side.
   b. Use measurement to justify that the angle you constructed is perpendicular to the side chosen. That is, justify that the line you constructed through the vertex is perpendicular to the side chosen.
   c. Use mathematics to justify this construction.
46. **CHALLENGE** Rewrite Theorem 3.9 in terms of two planes that are equidistant from a third plane. Sketch an example.

47. **WRITING IN MATH** Summarize the steps necessary to find the distance between a pair of parallel lines given the equations of the two lines.

48. **EXTENDED RESPONSE** Segment $AB$ is perpendicular to segment $CD$. Segment $AB$ and segment $CD$ bisect each other at point $X$.
   a. Draw a figure to represent the problem.
   b. Find $BD$ if $AB = 12$ and $CD = 16$.
   c. Find $BD$ if $AB = 24$ and $CD = 18$.

49. A city park is square and has an area of 81,000 square feet. Which of the following is the closest to the length of one side of the park?
   - F 100 ft
   - G 200 ft
   - H 300 ft
   - J 400 ft

50. **ALGEBRA** Pablo bought a sweater on sale for 25% off the original price and another 40% off the discounted price. If the sweater originally cost $48, what was the final price of the sweater?
   - A $14.40
   - B $21.60
   - C $31.20
   - D $36.00

51. **SAT/ACT** After $N$ cookies are divided equally among 8 children, 3 remain. How many would remain if $(N + 6)$ cookies were divided equally among the 8 children?
   - F 0
   - G 1
   - H 2
   - J 4

52. Refer to the figure at the right. Determine whether $a \parallel b$. Justify your answer.

![Figure](image)

Write an equation in point-slope form of the line having the given slope that contains the given point.

53. $m: \frac{1}{4}, (3, -1)$

54. $m: 0, (-2, 6)$

55. $m: -1, (-2, 3)$
56. \( m: -2, (-6, -7) \)

Prove the following.

57. If \( AB = BC \), then \( AC = 2BC \).

58. Given: \( JK \cong KL, HJ \cong GH, KL \cong HJ \)
Prove: \( GH \cong JK \)

59. MAPS Darnell sketched a map for his friend of the cross streets nearest to his home. Describe two different angle relationships between the streets.

Use the Distance Formula to find the distance between each pair of points.

60. \( A(0, 0), B(15, 20) \)

61. \( O(-12, 0), P(-8, 3) \)

62. \( C(11, -12), D(6, 2) \)

63. \( R(-2, 3), S(3, 15) \)

64. \( M(1, -2), N(9, 13) \)

65. \( Q(-12, 2), T(-9, 6) \)
Study Guide and Review - Vocabulary Check - Chapter 3

State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. If \( \angle 1 \cong \angle 5 \), then lines \( p \) and \( q \) are **skew** lines.
2. Angles 4 and 6 are **alternate** interior angles.
3. Angles 1 and 7 are alternate **exterior** angles.
4. If lines \( p \) and \( q \) are parallel, then angles 3 and 6 are **congruent**.
5. The distance from point \( X \) to line \( q \) is the length of the segment **perpendicular** to line \( q \) from \( X \).
6. Line \( t \) is called the **transversal** for lines \( p \) and \( q \).
7. If \( p \parallel q \), then \( \angle 2 \) and \( \angle 8 \) are **supplementary**.
8. Angles 4 and 8 are **corresponding** angles.
Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

9. $\angle 1$ and $\angle 5$

10. $\angle 4$ and $\angle 6$

11. $\angle 2$ and $\angle 8$

12. $\angle 4$ and $\angle 5$

13. **BRIDGES** The Roebling Suspension Bridge extends over the Ohio River connecting Cincinnati, Ohio, to Covington, Kentucky. Describe the type of lines formed by the bridge and the river.
Name:

In the figure, \( m \angle 1 = 123 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

14. \( \angle 5 \)
15. \( \angle 14 \)
16. \( \angle 16 \)
17. \( \angle 11 \)
18. \( \angle 4 \)
19. \( \angle 6 \)

20. **MAPS** The diagram shows the layout of Elm, Plum, and Oak streets. Find the value of \( x \).

Determine whether \( \overline{AB} \) and \( \overline{XY} \) are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

21. \( A(5, 3), B(8, 0), X(7, -2), Y(1, 10) \)
22. \( A(-3, 9), B(0, 7), X(4, 13), Y(-5, 7) \)
23. \( A(8, 1), B(-2, 7), X(-6, 2), Y(-1, -1) \)

Graph the line that satisfies each condition.

24. contains \((-3, 4)\) and is parallel to \( \overline{AB} \) with \( A(2, 5) \) and \( B(9, 2) \)

25. contains \((1, 3)\) and is perpendicular to \( \overline{PQ} \) with \( P(4, -6) \) and \( Q(6, -1) \)

26. **AIRPLANES** Two Oceanic Airlines planes are flying at the same altitude. Using satellite imagery, each plane’s position can be mapped onto a coordinate plane. Flight 815 was mapped at \((23, 17)\) and \((5, 11)\) while Flight 44 was mapped at \((3, 15)\) and \((9, 17)\). Determine whether their paths are *parallel*, *perpendicular*, or *neither*. 
Write an equation in point-slope form of the line having the given slope that contains the given point.

27. \( m = 2, \ (4, -9) \)

28. \( m = -\frac{3}{4}, \ (8, -1) \)

Write an equation in slope-intercept form of the line having the given slope and \( y \)-intercept.

29. \( m = 5, \ y \)-intercept: \(-3\)

30. \( m = \frac{1}{2}, \ y \)-intercept: \(4\)

Write an equation in slope-intercept form for each line.

31. \((-3, 12)\) and \((15, 0)\)

32. \((-7, 2)\) and \((5, 8)\)

33. **WINDOW CLEANING** Ace Window Cleaning Service charges \$50 for the service call and \$20 for each hour spent on the job. Write an equation in slope-intercept form that represents the total cost \( C \) in terms of the number of hours \( h \).

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

34. \( \angle 7 \equiv \angle 10 \)

35. \( \angle 2 \equiv \angle 10 \)

36. \( \angle 1 \equiv \angle 3 \)

37. \( \angle 3 \equiv \angle 11 \)

38. Find \( x \) so that \( p \parallel q \). Identify the postulate or theorem you used.
39. **LANDSCAPING** Find the measure needed for $m \angle ADC$ that will make $\overline{AB} \parallel \overline{CD}$ if $m \angle BAD = 45$.

Copy each figure. Draw the segment that represents the distance indicated.

40. $X$ to $\overline{VW}$

41. $L$ to $\overline{JK}$

42. **HOME DÉCOR** Scott wants to hang two rows of framed pictures in parallel lines on his living room wall. He first spaces the nails on the wall in a line for the top row. Next, he hangs a weighted plumb line from each nail and measures an equal distance below each nail for the second row. Why does this ensure that the two rows of pictures will be parallel?
Chapter 3 - Parallel and Perpendicular Lines - Practice
Test - Chapter 3

Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles.*

1. \( \angle 6 \) and \( \angle 3 \)
2. \( \angle 4 \) and \( \angle 7 \)
3. \( \angle 5 \) and \( \angle 4 \)

Determine the slope of the line that contains the given points.

4. \( G(8, 1), H(8, -6) \)
5. \( A(0, 6), B(4, 0) \)
6. \( E(6, 3), F(-6, 3) \)
7. \( E(5, 4), F(8, 1) \)

In the figure, \( m \angle 8 = 96 \) and \( m \angle 12 = 42 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

8. \( \angle 9 \)
9. \( \angle 11 \)
10. \( \angle 6 \)

11. Find the value of the variable in the figure below.

12. **FITNESS** You would like to join a fitness center. Fit-N-Trim charges $80 per month. Fit-For-Life charges a membership fee of $200 and $75 per month. For how many months would the total cost be the same?
Write an equation in slope-intercept form for each line described.

13. passes through (−8, 1), perpendicular to \( y = 2x - 17 \)

14. passes through (0, 7), parallel to \( y = 4x - 19 \)

15. passes through (−12, 3), perpendicular to \( y = \frac{2}{3}x - 11 \)

Find the distance between each pair of parallel lines with the given equations.

16. \( y = x - 11 \)
   \( y = x - 7 \)

17. \( y = -2x + 1 \)
   \( y = -2x + 16 \)

18. MULTIPLE CHOICE Which segment is skew to \( \overline{CD} \)?

   A \( \overline{ZY} \)
   B \( \overline{AB} \)
   C \( \overline{DE} \)
   D \( \overline{VZ} \)

19. Find \( x \) so that \( a \parallel b \). Identify the postulate or theorem you used.

   \( a \) \( \overrightarrow{b} \)
   \( (4x + 11)^\circ \) \( (8x + 1)^\circ \)

COORDINATE GEOMETRY Find the distance from \( P \) to \( \ell \).

20. Line \( \ell \) contains points (−4, 2) and (3, −5). Point \( P \) has coordinates (1, 2).

21. Line \( \ell \) contains points (6, 5) and (2, 3). Point \( P \) has coordinates (2, 6).

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.
22. $\angle 4 \cong \angle 10$

23. $\angle 9 \cong \angle 6$

24. $\angle 7 \cong \angle 11$

25. **JOBS** Hailey works at a gift shop after school. She is paid $10 per hour plus a 15% commission on merchandise she sells. Write an equation that represents her earnings in a week if she sold $550 worth of merchandise.

**Chapter 3 - Parallel and Perpendicular Lines - Preparing for Standardized Tests - Chapter 3**

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. **GRIDDED RESPONSE** What is the slope of the line that contains the points $R(-2, 1)$ and $S(10, 6)$? Express your answer as a fraction.

2. **GRIDDED RESPONSE** Solve for $x$ in the figure below.

![Diagram with angles](image)
Chapter 3 - Parallel and Perpendicular Lines - Standardized Test Practice - Cumulative, Chapters 1-3

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. If \(a \parallel b\) in the diagram below, which of the following may not be true?

   ![Diagram]

   A \( \angle 1 \equiv \angle 3 \)
   B \( \angle 4 \equiv \angle 7 \)
   C \( \angle 2 \equiv \angle 5 \)
   D \( \angle 8 \equiv \angle 2 \)

2. Which of the following best describes a *counterexample* to the assertion below?

   *The sum of two odd numbers is odd.*

   F \( 3 + 3 = 6 \)
   G \( 5 + 4 = 9 \)
   H \( 6 + 2 = 8 \)
   J \( 4 + 9 = 13 \)
3. What is the slope of the line?

\[ \frac{2}{3} \]

A \( \frac{2}{3} \)

B \( \frac{1}{2} \)

C \( \frac{2}{5} \)

D \( \frac{1}{6} \)

4. Line \( k \) contains points at (4, 1) and (-5, -5). Find the distance between line \( k \) and point \( F(-4, 0) \).

F 3.3 units

G 3.6 units

H 4.0 units

J 4.2 units

5. The globe has a diameter of 22 inches. What is the volume of the globe?

A 1520.5 in\(^3\)

B 1741.4 in\(^3\)

C 5575.3 in\(^3\)

D 6014.8 in\(^3\)

6. What is \( m \angle 1 \) in the figure below?

F 85

G 90

H 95

J 100
7. Jason is saving money to buy a car stereo. He has $45 saved, and he can save $15 per week. If the stereo that he wants is $210, how many weeks will it take Jason to buy the stereo?
   A 10
   B 11
   C 12
   D 13

   Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** For a given line and a point not on the line, how many lines exist that pass through the point and are parallel to the given line?

9. **GRIDDED RESPONSE** Find the slope of the line that contains the points (4, 3) and (−2, −5).

10. Use the proof to answer the question.
    Given: \(\angle 1 \equiv \angle 2\)
    Prove: \(a \parallel b\)

    ![Diagram]

    Proof:

    | Statements | Reasons |
    |------------|---------|
    | 1. \(\angle 1 \equiv \angle 2\) | 1. Given |
    | 2. \(\angle 2 \equiv \angle 3\) | 2. ? |
    | 3. \(\angle 1 \equiv \angle 3\) | 3. Transitive Prop. |
    | 4. \(a \parallel b\) | 4. If corresponding angles are congruent, then the lines are parallel. |

11. Write the contrapositive of the statement.
    *If a figure is a square, then the figure is a parallelogram.*

    Record your answers on a sheet of paper. Show your work.

12. Refer to the figure to identify each of the following.

    ![Diagram]

    a. all segments parallel to \(\overline{MQ}\)
    b. all planes intersecting plane \(SRN\)
    c. a segment skew to \(\overline{ON}\)
13. Use this graph to answer each question.
   a. What is the equation of line $m$?
   b. What is the slope of a line that is parallel to line $m$?
   c. What is the slope of a line that is perpendicular to line $m$?