Chapter 9 - Transformations and Symmetry - Get Ready for Chapter 9

Identify the type of congruence transformation shown as a reflection, translation, or rotation.

1. A to B
2. D to A
3. A to C

Find the sum of each pair of vectors.

4. \((13, -4) + (-11, 9)\)
5. \((6, -31) + (-22, 3)\)

6. BAND During part of a song, the drummer in a marching band moves from \((1, 4)\) to \((5, 1)\). Write the component form of the vector that describes his movement.

Determine whether the dilation from \(A\) to \(B\) is an enlargement or a reduction. Then find the scale factor of the dilation.

7.

8. PLAYS Bob is making a model of an ant for a play. Find the scale factor of the model if the ant is \(\frac{1}{2}\) inch long and the model is 1 foot long.
9-1 Reflections - Check Your Understanding

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

1. 

2. 

3. 

4. **SPORTING EVENTS** Toru is waiting at a café for a friend to bring him a ticket to a sold-out sporting event. At what point $P$ along the street should the friend try to stop his car to minimize the distance Toru will have to walk from the café, to the car, and then to the arena entrance? Draw a diagram.

Graph $\triangle ABC$ and its image in the given line.

5. $y = -2$
7. \( \triangle XYZ \) with vertices \( X(0, 4) \), \( Y(-3, 4) \), and \( Z(-4, -1) \) in the \( y \)-axis

8. \( \triangle QRS \) with vertices \( Q(-1, 4) \), \( R(4, 4) \), \( S(3, 1) \), and \( T(-2, 1) \) in the \( x \)-axis

9. quadrilateral \( JKLM \) with vertices \( J(-3, 1) \), \( K(-1, 3) \), \( L(1, 3) \), and \( M(-3, -1) \) in the line \( y = x \)
9-1 Reflections - Practice and Problem Solving

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

10. 

11. 

12. 

13. 

14.
15. **SPORTS** When a ball is rolled or struck without spin against a wall, it bounces off the wall and travels in a ray that is the reflected image of the path of the ball if it had gone straight through the wall. Use this information in Exercises 16 and 17.

16. **BILLIARDS** Tadeo is playing billiards. He wants to strike the eight ball with the cue ball so that the eight ball bounces off the rail and rolls into the indicated pocket. If the eight ball moves with no spin, draw a diagram showing the exact point \( P \) along the right rail where the eight ball should hit after being struck by the cue ball.

17. **INDOOR SOCCER** Abby is playing indoor soccer, and she wants to hit the ball to point \( C \), but must avoid an opposing player at point \( B \). She decides to hit the ball at point \( A \) so that it bounces off the side wall. Draw a diagram that shows the exact point along the top wall for which Abby should aim.
Graph each figure and its image in the given line.

18. \( \triangle ABC; \ y = 3 \)
19. \( \triangle ABC; \ x = -1 \)
20. \( JKLM; \ x = 1 \)
21. \( JKLM; \ y = 4 \)
22. \( WXYZ; \ y = -4 \)
23. \( WXYZ; \ x = -2 \)

Graph each figure and its image under the given reflection.

24. rectangle \( ABCD \) with vertices \( A(-5, 2), B(1, 2), C(1, -1), \) and \( D(-5, -1) \) in the line \( y = -2 \)
25. square \( JKLM \) with vertices \( J(-4, 6), K(0, 6), L(0, 2), \) and \( M(-4, 2) \) in the \( y \)-axis
26. \( \triangle FGH \) with vertices \( F(-3, 2), G(-4, -1), \) and \( H(-6,-1) \) in the line \( y = x \)
27. \( \triangle WXYZ \) with vertices \( W(2, 3), X(7, 3), Y(6, -1), \) and \( Z(1, -1) \) in the \( x \)-axis
28. trapezoid $PQRS$ with vertices $P(-1, 4), Q(2, 4), R(1, -1)$, and $S(-1, -1)$ in the $y$-axis

29. $\triangle FGH$ with vertices $S(-3, -2), T(-2, 3)$, and $U(2, 2)$ in the line $y = x$

Each figure shows a preimage and its reflected image in some line. Copy each figure and draw the line of reflection.

30. 

31. 

32. 

CONSTRUCTION To construct the reflection of a figure in a line using only a compass and a straightedge, you can use:
- the construction of a line perpendicular to a given line through a point not on the line (p. 55), and
- the construction of a segment congruent to a given segment (p. 17).

Copy each figure and the given line of reflection. Then construct the reflected image.
33.

34.

35. PHOTOGRAPHY Refer to the photo at the right.
   a. What object separates the zebras and their reflections?
   b. What geometric term can be used to describe this object?

![Zebras](image)

ALGEBRA Graph the line \( y = 2x - 3 \) and its reflected image in the given line. What is the equation of the reflected image?

36. \( x \)-axis

37. \( y \)-axis

38. \( y = x \)

39. Reflect \( \triangle CDE \) shown below in the line \( y = 3x \).
40. Relocate vertex $C$ so that $ABCDE$ the line $y = 3x$ is convex, and all sides remain the same length.

ALGEBRA Graph the reflection of each function in the given line. Then write the equation of the reflected image.

41. $x$-axis

42. $y$-axis

43. $x$-axis
44. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a reflection in the origin.
   a. **GEOMETRIC** Draw $\triangle ABC$ in the coordinate plane so that each vertex is a whole-number ordered pair.
   b. **GRAPHICAL** Locate each reflected point $A', B'$, and $C'$ so that the reflected point, the original point, and the origin are collinear, and both the original point and the reflected point are equidistant from the origin.
   c. **TABULAR** Copy and complete the table below.

```
<table>
<thead>
<tr>
<th>Coordinates</th>
<th>$\triangle ABC$</th>
<th>$\triangle A'B'C'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>A'</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>B'</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>C</td>
</tr>
</tbody>
</table>
```

d. **VERBAL** Make a conjecture about the relationship between corresponding vertices of a figure reflected in the origin.

45. **FIND THE ERROR** Jamil and Ashley are finding the coordinates of the image of $(2, 3)$ after a reflection in the $x$-axis. Is either of them correct? Explain.

```
Jamil
C'(2, -3)

Ashley
C'(-2, 3)
```

46. **WRITING IN MATH** Describe how to reflect a figure not on the coordinate plane across a line.

47. **CHALLENGE** A point in the second quadrant with coordinates $(-a, b)$ is reflected in the $x$-axis. If the reflected point is then reflected in the line $y = -x$, what are the final coordinates of the image?

48. **OPEN ENDED** Draw a polygon on the coordinate plane that when reflected in the $x$-axis looks exactly like the original figure.

49. **CHALLENGE** When $A(4, 3)$ is reflected in a line, its image is $A'(-1, 0)$. Find the equation of the line of reflection. Explain your reasoning.

50. **REASONING** The image of a point reflected in a line is *sometimes, always, or never* located on the other side of the line of reflection.

51. **WRITING IN MATH** Suppose points $P$, $Q$, and $R$ are collinear, with point $Q$ between points $P$ and $R$. Describe a plan for a proof that the reflection of points $P$, $Q$, and $R$ in a line preserves collinearity and betweenness of points.

52. **SHORT RESPONSE** If quadrilateral $WXYZ$ is reflected across the $y$-axis to become quadrilateral $W'X'Y'Z'$ what are the coordinates of $X'$?
Name:

53. **ALGEBRA** If the arithmetic mean of $6x$, $3x$, and 27 is 18, then what is the value of $x$?
   A 2  
   B 3  
   C 5  
   D 6

54. In $\triangle DEF$, $m\angle E = 108^\circ$, $m\angle F = 26^\circ$, and $f = 20$. Find $d$ to the nearest whole number.

   [Diagram]
   F 26  
   G 33  
   H 60  
   J 65

55. **SAT/ACT** In a coordinate plane, points $A$ and $B$ have coordinates $(-2, 4)$ and $(3, 3)$, respectively. What is the value of $AB$?
   A $(1, 7)$  
   B $\sqrt{26}$  
   C $(5, -1)$  
   D $\sqrt{50}$

   Write the component form of each vector.

56. [Diagram]

57. [Diagram]

58. [Diagram]
59. **REAL ESTATE** A house is built on a triangular plot of land. Two sides of the plot are 160 feet long, and they meet at an angle of 85°. If a fence is to be placed along the perimeter of the property, how much fencing material is needed?

![Diagram of a triangular plot of land with sides of 160 feet and an angle of 85°.]

60. **COORDINATE GEOMETRY** In \( \triangle LMN \), \( PR \) divides \( NL \) and \( MN \) proportionally. If the vertices are \( N(8, 20) \), \( P(11, 16) \), and \( R(3, 8) \) and \( \frac{LP}{PN} = \frac{2}{1} \), find the coordinates of \( L \) and \( M \).

Use the figure at the right to write an inequality relating the given pair of angle or segment measures.

![Diagram of a polygon with labeled segments and angles.]

61. \( AB, FD \)

62. \( m\angle BDC, m\angle FDB \)

63. \( m\angle FBA, m\angle DBF \)

**Find the magnitude and direction of each vector.**

64. \( \overrightarrow{RS} : R(-3, 3) \) and \( S(-9, 9) \)

65. \( \overrightarrow{JK} : J(8, 1) \) and \( K(2, 5) \)

66. \( \overrightarrow{FG} : F(-4, 0) \) and \( G(-6, -4) \)

67. \( \overrightarrow{AB} : A(-1, 10) \) and \( B(1, -12) \)
9-2 Translations - Check Your Understanding

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

1. 

2. 

3. 

Graph each figure and its image along the given vector.

4. trapezoid $JKLM$ with vertices $J(2, 4)$, $K(1, 1)$, $L(5, 1)$, and $M(4, 4)$; $\langle 7,1 \rangle$

5. $\triangle DFG$ with vertices $D(-8, 8)$, $F(-10, 4)$, and $G(-7, 6)$; $\langle 5,-2 \rangle$

6. parallelogram $WXYZ$ with vertices $W(-6, -5)$, $X(-2, -5)$, $Y(-1, -8)$, and $Z(-5, -8)$; $\langle -1,4 \rangle$

7. **VIDEO GAMES** The object of the video game shown is to manipulate the colored tiles left or right as they fall from the top of the screen to completely fill each row without leaving empty spaces. If the starting position of the tile piece at the top of the screen is $(x, y)$, use function notation to describe the translation that will fill the indicated row.
9-2 Translations - Practice and Problem Solving

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

8.

9.

10.

11.

12.

13.

Graph each figure and its image along the given vector.
15. \(\triangle MNP\) with vertices \(M(4, -5), N(5, -8),\) and \(P(8, -6); \langle -2, 5 \rangle\)

16. rectangle \(QRST\) with vertices \(Q(-8, 4), R(-8, 2), S(-3, 2),\) and \(T(-3, 4); \langle 2, 3 \rangle\)

17. quadrilateral \(FGHI\) with vertices \(F(-4, -2), G(-1, -1), H(0, -4),\) and \(J(-3, -6); \langle -5, -2 \rangle\)

18. \(\varphi WXYZ\) with vertices \(W(-3, -1), X(1, -1), Y(2, -4),\) and \(Z(-2, -4); \langle -3, 4 \rangle\)

19. trapezoid \(JKLM\) with vertices \(J(-4, -2), K(-1, -2), L(0, -5),\) and \(M(-5, -5); \langle 6, 5 \rangle\)

20. **LOCATION** Brittany’s neighborhood is shown on the grid at the right.

   ![Grid Image]

   a. If she leaves home and travels 4 blocks north and 3 blocks east, what is her new location?
   b. Use words to describe two possible translations that will take Brittany home from school.

21. **FOOTBALL** A wide receiver starts from the 15-yard line on the right hash mark and runs a route that takes him 12 yards to the left and down field for a gain of 17 yards. Write a translation vector to describe the receiver’s route.

   ![Football Field Diagram]
22. **CHESS** Each chess piece has a path that it can follow to move. The rook, which begins in square a8, can only move vertically or horizontally. The knight, which begins in square b8, can move two squares horizontally and then one square vertically, or two squares vertically and one square horizontally. The bishop, which begins in square f8, can only move diagonally.

![Chessboard](image)

a. The knight moves 2 squares vertically and 1 square horizontally on its first move, then two squares horizontally and 1 square vertically on its second move. What are the possible locations for the knight after two moves?

b. After two moves, the rook is in square d3. Describe a possible translation to describe the two moves.

c. Describe a translation that can take the bishop to square a1. What is the minimum number of moves that can be used to accomplish this translation?

Write each translation vector.

23.

![Translation Vector Diagram](image)

24.

![Translation Vector Diagram](image)

25. **CONCERTS** Dexter’s family buys tickets every year for a concert. Last year they were in seats C3, C4, C5, and C6. This year, they will be in seats D16, D17, D18, and D19. Write a translation in words and using vector notation that can be used to describe the change in their seating.

![Seating Chart](image)
ALGEBRA Graph the translation of each function along the given vector. Then write the equation of the translated image.

26. $\langle 4,1 \rangle$

27. $\langle -2,0 \rangle$

28. **ROLLER COASTERS** The length of the roller coaster track from the top of a hill to the bottom of the hill is 125 feet at a $53^\circ$ angle with the vertical. If the position at the top of the hill is $(x, y)$, use function notation to describe the translation to the bottom of the hill. Round to the nearest foot.

29. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate reflections over a pair of parallel lines.

a. **GEOMETRIC** On patty paper, draw $\triangle ABC$ and a pair of vertical lines $l$ and $m$. Reflect in line $l$ by folding the patty paper. Then reflect, $\triangle A'B'C'$, in line $m$. Label the final image $\triangle A''B''C''$.

b. **GEOMETRIC** Repeat the process in part a for $\triangle DEF$ reflected in vertical lines $n$ and $p$ and $\triangle MNP$ reflected in vertical lines $q$ and $r$.

c. **TABULAR** Copy and complete the table below.

<table>
<thead>
<tr>
<th>Distance Between Corresponding Points (cm)</th>
<th>Distance Between Vertical Lines (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ and $A''$, $B$ and $B''$, $C$ and $C''$</td>
<td>$l$ and $m$</td>
</tr>
<tr>
<td>$D$ and $D''$, $E$ and $E''$, $F$ and $F''$</td>
<td>$n$ and $p$</td>
</tr>
<tr>
<td>$M$ and $M''$, $N$ and $N''$, $P$ and $P''$</td>
<td>$q$ and $r$</td>
</tr>
</tbody>
</table>

d. **VERBAL** Describe the result of two reflections in two vertical lines using one transformation.
30. **REASONING** Determine a rule to find the final image of a point that is translated along \((x + a, y + b)\) and then \((x + c, y + d)\).

31. **CHALLENGE** A line \(y = mx + b\) is translated using the vector \((a, b)\). Write the equation of the translated line. What is the value of the \(y\)-intercept?

32. **OPEN ENDED** Draw a figure on the coordinate plane so that the figure has the same orientation after it is reflected in the line \(y = 1\). Explain what must be true in order for this to occur.

33. **WRITING IN MATH** Compare and contrast function notation and vector notation for translations.

34. **WRITING IN MATH** Recall from Lesson 9-1 that an invariant point maps onto itself. Can invariant points occur with translations? Explain why or why not.

35. Identify the location of point \(P\) under translation \((x + 3, y + 1)\).

![Diagram](image1.png)

A (0, 6)
B (0, 3)
C (2, -4)
D (2, 4)

36. **SHORT RESPONSE** Which vector best describes the translation of \(A(3, -5)\) to \(A'(-2, -8)\) ?

37. **ALGEBRA** Over the next four days, Amanda plans to drive 160 miles, 235 miles, 185 miles, and 220 miles. If her car gets an average of 32 miles per gallon of gas, how many gallons of gas should she expect to use in all?

F 25
G 30
H 35
J 40

38. **SAT/ACT** A bag contains 5 red marbles, 2 blue marbles, 4 white marbles, and 1 yellow marble. If two marbles are chosen in a row, without replacement, what is the probability of getting 2 white marbles?

A. \(\frac{1}{6}\)
B. \(\frac{1}{11}\)
C. \(\frac{1}{9}\)
D. \(\frac{5}{33}\)

Graph each figure and its image under the given reflection.

39. **\(\overline{DJ}\) with endpoints \(D(4, 4), J(-3, 2)\) in the \(y\)-axis**
Name:

40. \(\triangle XYZ\) with vertices \(X(0, 0), Y(3, 0),\) and \(Z(0, 3)\) in the x-axis

41. \(\triangle ABC\) with vertices \(A(-3, -1), B(0, 2),\) and \(C(3, -2)\), in the line \(y = x\)

42. quadrilateral \(JKLM\) with vertices \(J(-2, 2), K(3, 1), L(4, -1),\) and \(M(-2, -2)\) in the origin

Copy the vectors to find each sum or difference.

43. \(\vec{c} - \vec{d}\)

44. \(\vec{w} + \vec{x}\)

45. \(\vec{n} - \vec{p}\)

46. NAVIGATION An airplane is three miles above sea level when it begins to climb at a 3.5\(^\circ\) angle. If this angle is constant, how far above sea level is the airplane after flying 50 miles?

Use \(\triangle JKL\) to find each measure.

47. \(m \angle MJK\)

48. \(m \angle JML\)

49. \(m \angle JKL\)

50. \(m \angle KJL\)
Copy the diagram shown, and extend each ray. Classify each angle as right, acute, or obtuse. Then use a protractor to measure the angle to the nearest degree.

Explore 9-3 Geometry Lab: Rotations - Exercises

1. Graph \( \triangle JKL \) with vertices \( J(1, 3) \), \( K(2, 1) \), and \( L(3, 4) \) on a coordinate plane, and then trace on patty paper.

   a. Use a protractor to rotate each vertex 90\(^\circ\) clockwise about the origin as shown in the figure at the right. What are the vertices of the rotated image?

   b. Rotate \( \triangle JKL \) 180\(^\circ\) about the origin. What are the vertices of the rotated image?

   c. Use the Distance Formula to find the distance from points \( J, K, \) and \( L \) to the origin. Repeat for \( J'K'L' \) and \( J''K''L'' \).

2. **WRITING IN MATH** If you rotate point \( (4, 2) \) 90\(^\circ\) and 180\(^\circ\) about the origin, how do the \( x \)- and \( y \)-coordinates change?

3. **MAKE A PREDICTION** What are the new coordinates of a point \( (x, y) \) that is rotated 270\(^\circ\)?

4. **MAKE A CONJECTURE** Make a conjecture about the distances from the center of rotation \( P \) to each corresponding vertex of \( ABCD \) and \( A'B'C'D' \).
9-3 Rotations - Check Your Understanding

Copy each polygon and point $K$. Then use a protractor and ruler to draw the specified rotation of each figure about point $K$.

1. $45^\circ$

2. $120^\circ$

3. Triangle $DFG$ has vertices $D(-2, 6)$, $F(2, 8)$, and $G(2, 3)$. Graph $\triangle DFG$ and its image after a rotation $180^\circ$ about the origin.
4. **MULTIPLE CHOICE** For the transformation shown, what is the measure of the angle of rotation of $ABCD$ about the origin?

![Diagram]

A $90^\circ$
B $180^\circ$
C $270^\circ$
D $360^\circ$

9-3 Rotations - Practice and Problem Solving

Copy each polygon and point $K$. Then use a protractor and ruler to draw the specified rotation of each figure about point $K$.

5. $90^\circ$

![Diagram]

6. $15^\circ$

![Diagram]

7. $145^\circ$

![Diagram]
8. \(30^\circ\)

9. \(260^\circ\)

10. \(50^\circ\)

PINWHEELS Find the angle of rotation to the nearest tenth of a degree that maps \(P\) onto. Explain your reasoning.
Graph each figure and its image after the specified rotation about the origin.

14. ΔJKL has vertices J(2, 6), K(5, 2), and L(7, 5); 90°

15. rhombus WXYZ has vertices W(−3, 4), X(0, 7), Y(3, 4), and Z(0, 1); 90°

16. ΔFGH has vertices F(2, 4), G(5, 6), and H(7, 2); 180°

17. trapezoid ABCD has vertices A(−7, −2), B(−6, −6), C(−1, −1), and D(−5, 0); 180°

18. ΔRST has vertices R(−6, −1), S(−4, −5), and T(−2, −1); 270°

19. parallelogram MPQV has vertices M(−6, 3), P(−2, 3), Q(−3, −2), and V(−7, −2); 270°

20. WEATHER A weathervane is used to indicate the direction of the wind. If the vane is pointing northeast and rotates 270°, what is the new wind direction?
Name:

21. PHOTOGRAPHY The photograph of the Grande Roue, or Big Wheel, at the left appears blurred because of the camera’s shutter speed—the length of time the camera’s shutter was open.

   ![Image of a Ferris wheel]

   a. Estimate the angle of rotation in the photo.
   (Hint: Use points A and A’. ) 10°
   b. If the Ferris wheel makes one revolution per minute, use your estimate from part a to estimate the camera’s shutter speed.

   Each figure shows a preimage and its image after a rotation about point P. Copy each figure, locate point P, and find the angle of rotation.

22. ![Image of a rotated figure]

23. ![Image of two rotated figures]

   ALGEBRA Give the equation of the line \( y = -x - 2 \) after a rotation about the origin through the given angle. Then describe the relationship between the equations of the image and preimage.

   ![Image of a graph with a line and a rotated image]

   24. 90°

   25. 180°

   26. 270°

   27. 360°
ALGEBRA Rotate the line the specified number of degrees about the \( x \)- and \( y \)-intercepts and find the equation of the resulting image.

28. \( y = x - 5; 90^\circ \)

29. \( y = 2x + 4; 180^\circ \)

30. \( y = 3x - 2; 270^\circ \)

31. RIDES An amusement park ride consists of four circular cars. The ride rotates at a rate of 0.25 revolution per second. In addition, each car rotates 0.5 revolution per second. If Jane is positioned at point \( P \) when the ride begins, what coordinates describe her position after 31 seconds?

![Diagram of an amusement park ride with coordinates and a point labeled P.]

32. BICYCLE RACING Brandon and Nestor are participating in a bicycle race on a circular track with a radius of 200 feet.

![Diagram of a bicycle race with a point \((x, y)\) and a circle labeled with radius 200.]

a. If the race starts at \((200, 0)\) and both complete one rotation in 30 seconds, what are their coordinates after 5 seconds?

b. Suppose the length of race is 50 laps and Brandon continues the race at the same rate. If Nestor finishes in 26.2 minutes, who is the winner?

MULTIPLE REPRESENTATIONS In this problem, you will investigate reflections over a pair of intersecting lines.
33. a. **GEOMETRIC** On a coordinate $y$ plane, draw a triangle and a pair of intersecting lines. Label the triangle $ABC$ and the lines $l$ and $m$. Reflect $\triangle ABC$ in the line $l$. Then reflect $\triangle A'B'C''$ in the line $m$. Label the final image $A''B''C''$.

![Diagram of geometric transformation]

b. **GEOMETRIC** Repeat the process in part a two more times in two different quadrants. Label the second triangle $DEF$ and reflect it in intersecting lines $n$ and $p$. Label the third triangle $MNP$ and reflect it in intersecting lines $q$ and $r$.

c. **TABULAR** Measure the angle of rotation of each triangle about the point of intersection of the two lines. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Angle of Rotation Between Figures</th>
<th>Angle Between Intersecting Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ABC$ and $\triangle A''B''C''$</td>
<td>$\ell$ and $m$</td>
</tr>
<tr>
<td>$\triangle DEF$ and $\triangle D''E''F'$</td>
<td>$n$ and $p$</td>
</tr>
<tr>
<td>$\triangle MNP$ and $\triangle M''N''P''$</td>
<td>$q$ and $r$</td>
</tr>
</tbody>
</table>

d. **VERBAL** Make a conjecture about the angle of rotation of a figure about the intersection of two lines after the figure is reflected in both lines.

34. **WRITING IN MATH** Are collinearity and betweenness of points maintained under rotation? Explain.

35. **CHALLENGE** Point $C$ has coordinates $(5, 5)$. The image of this point after a rotation of $100^\circ$ about a certain point is $C'(-5, 7.5)$. Find the coordinates of the centre of this rotation. Explain.

36. **OPEN ENDED** Draw a figure on the coordinate plane. Describe a nonzero rotation that maps the image onto the preimage with no change in orientation.

37. **REASONING** Is the reflection of a figure in the $x$-axis equivalent to the rotation of that same figure $180^\circ$ about the origin? Explain.

38. **WRITING IN MATH** Do invariant points sometimes, always, or never occur in a rotation? Explain your reasoning.
39. What rotation of trapezoid $QRST$ creates an image with point $R'$ at (4, 3)?

A 270° counterclockwise about point $T$
B 185° counterclockwise about point $T$
C 180° clockwise about the origin
D 90° clockwise about the origin

40. **SHORT RESPONSE** $\triangle XYZ$ has vertices $X(1, 7)$, $Y(0, 2)$, and $Z(-5, -2)$. What are the coordinates of $X'$ after a rotation 270° counterclockwise about the origin?

41. **ALGEBRA** The population of the United States in July of 2007 was estimated to have surpassed 301,000,000. At the same time the world population was estimated to be over 6,602,000,000. What percent of the world population, to the nearest tenth, lived in the United States at this time?

F 3.1%
H 4.2%
G 3.5%
J 4.6%

42. **SAT/ACT** An 18-foot ladder is placed against the side of a house. The base of the ladder is positioned 8 feet from the house. How high up on the side of the house, to the nearest tenth of a foot, does the ladder reach?

A 10.0 ft
B 16.1 ft
C 19.7 ft
D 26.0 ft

43. **VOLCANOES** A cloud of dense gas and dust from a volcano blows 40 miles west and then 30 miles north. Make a sketch to show the translation of the dust particles. Then find the distance of the shortest path that would take the particles to the same position

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

44.
45. Identify the type of congruence transformation shown as a reflection, translation, or rotation.

46. 

47. 

48. 

49. 
Chapter 9 - Transformations and Symmetry - Mid-Chapter
Quiz: Lessons 9-1 through 9-3

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

1.

2.

Graph each figure and its image after the specified reflection.

3. $\triangle FGH$ has vertices $F(-4, 3)$, $G(-2, 0)$, and $H(-1, 4)$; in the y-axis

4. rhombus $QRST$ has vertices $Q(2, 1)$, $R(4, 3)$, $S(6, 1)$, and $T(4, -1)$; in the x-axis

5. CLUBS The drama club is selling candy during the intermission of a school play. Locate point $P$ along the wall to represent the candy table so that people coming from either door $A$ or door $B$ would walk the same distance to the table.

Graph each figure and its image after the specified translation.

6. $\triangle ABC$ with vertices $A(0, 0)$, $B(2, 1)$, $C(1, -3)$; $\langle 3, -1 \rangle$

7. rectangle $JKLM$ has vertices $J(-4, 2)$, $K(-4, -2)$, $L(-1, -2)$, and $M(-1, 2)$; $\langle 5, -3 \rangle$

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

8.
9.

10. **COMICS** Alex is making a comic. He uses graph paper to make sure the dimensions of his drawings are accurate. If he draws a coordinate plane with two flies as shown below, what vector represents the movement from fly 1 to fly 2?

   ![Coordinate Plane with Flies](image1.png)

   Copy each polygon and point \( R \). Then use a protractor and ruler to draw the specified rotation of each figure about point \( R \).

11. **45°**

   ![Polygon with Point R](image2.png)

12. **60°**

   ![Rectangle with Point R](image3.png)

13. **MULTIPLE CHOICE** What is the image of point \( M \) after a rotation of 90° about the origin?

   ![Rotation Around Origin](image4.png)

   - A \((-3, 1)\)
   - B \((-3, -1)\)
   - C \((-1, -3)\)
   - D \((3, 1)\)
Graph each figure and its image after the specified rotation.

14. $\triangle RST$ has vertices $R(-3, 0), S(-1, -4)$, and $T(0, -1)$; $90^\circ$

15. square $JKLM$ has vertices $J(-1, 2), K(-1, -2), L(3, -2)$, and $M(3, 2)$; $180^\circ$
Explore 9-4 Geometry Software Lab: Compositions of Transformations
- Analyze the Results

1. How are the original figure and the final figure related?

2. What single transformation could be used to produce the final figure?

3. If you move line \( m \), what happens? If you move line \( r \)?

4. **MAKE A CONJECTURE** If you reflected the figure in a third line, what single transformation do you think could be used to produce the final figure? Explain your reasoning.

5. Repeat the activity for a pair of perpendicular lines. What single transformation could be used to produce the same final figure?

6. **MAKE A CONJECTURE** If you reflected the figure from Exercise 5 in a third line perpendicular to the second line, what single transformation do you think could be used to produce the final figure? Explain your reasoning.
9-4 Compositions of Transformations - Check Your Understanding

Triangle $CDE$ has vertices $C(-5, -1), D(-2, -5)$, and $E(-1, -1)$. Graph $\triangle CDE$ and its image after the indicated glide reflection.
1. Translation: along \( \langle 4, 0 \rangle \) Reflection: in \( x \)-axis

2. Translation: along \( \langle 0, 6 \rangle \) Reflection: in \( y \)-axis

3. The endpoints of \( JK \) are \( J(2, 5) \) and \( K(6, 5) \). Graph \( JK \) and its image after a reflection in the \( x \)-axis and a rotation \( 90^\circ \) about the origin.

   Copy and reflect figure \( S \) in line \( m \) and then line \( p \). Then describe a single transformation that maps \( S \) onto \( S'' \).

4. 

5. 

6. **TILE PATTERNS** Viviana is creating a pattern for the top of a table with tiles in the shape of isosceles triangles. Describe the transformation combination that was used to create the pattern.
9-4 Compositions of Transformations - Practice and Problem Solving

Graph each figure with the given vertices and its image after the indicated glide reflection.

7. \( \triangle RST : R(1, -4), S(6, -4), T(5, -1) \) Translation: along \( \langle 2, 0 \rangle \) Reflection: in x-axis

8. \( \triangle JKL : J(1, 3), K(5, 0), L(7, 4) \) Translation: along \( \langle -3, 0 \rangle \) Reflection: in x-axis
Name:

9. \( \triangle XYZ : X(-7, 2), Y(-5, 6), Z(-2, 4) \) 
   Translation: along \( \langle 0, -1 \rangle \) Reflection: in y-axis

10. \( \triangle ABC : A(2, 3), B(4, 7), C(7, 2) \) 
    Translation: along \( \langle 0, 4 \rangle \) Reflection: in y-axis

11. \( \triangle DFG : D(2, 8), F(1, 2), G(4, 6) \) 
    Translation: along \( \langle 3, 3 \rangle \) Reflection: in \( y = x \)

12. \( \triangle MPQ : M(-4, 3), P(-5, 8), Q(-1, 6) \) 
    Translation: along \( \langle -4, -4 \rangle \) Reflection: in \( y = x \)

Graph each figure with the given vertices and its image after the indicated composition of transformations.

13. \( 
\overline{WX} : W(-4, 6) \) and \( X(-4, 1) \) 
   Reflection: in x-axis Rotation: \( 90^\circ \) about origin

14. \( 
\overline{AB} : A(-3, 2) \) and \( B(3, 8) \) Rotation: \( 90^\circ \) about origin Translation: along \( \langle 4, 4 \rangle \)

15. \( 
\overline{FG} : F(1, 1) \) and \( G(6, 7) \) Reflection: in x-axis Rotation: \( 180^\circ \) about origin

16. \( 
\overline{RS} : R(2, -1) \) and \( S(6, -5) \) Translation: along \( \langle -2, -2 \rangle \) Reflection: in y-axis

Copy and reflect figure \( D \) in line \( m \) and then line \( p \). Then describe a single transformation that maps \( D \) onto \( D^* \).

17. 

18. 

\[ 
\begin{align*} 
&\text{Copy and reflect figure } D \text{ in line } m \text{ and then line } p. 
\text{Then describe a single transformation that maps } D \text{ onto } D^*. \end{align*} 
\]
Describe the transformations combined to create the outlined kimono pattern.
23.

24. **SKATEBOARDS** Elizabeth has airbrushed the pattern shown onto her skateboard. What combination of transformations did she use to create the pattern?

   ![Skateboard Pattern]

   **ALGEBRA** Graph each figure and its image after the indicated transformations.

25. Rotation: 90° about the origin Reflection: in x-axis

   ![Graph 1]


   ![Graph 2]

27. Find the coordinates of \( \Delta A'B'C' \) after a reflection in the x-axis and a rotation of 180° about the origin if \( \Delta ABC \) has vertices \( A(-3, 1), B(-2, 3), \) and \( C(-1, 0) \).
28. **FIGURE SKATING** Kayla is practicing her figure skating routine. What combination of transformations is needed for Kayla to start at $A$, skate to $A'$, and end up at $A''$?

![Figure Skating Diagram]

29. **DANCING** Describe the transformations combined to go from Step 1 to Step 3.

![Dancing Diagram]

30. **PROOF** Write a paragraph proof for one case of the Composition of Isometries Theorem.
   Given: A translation along $\langle a, b \rangle$ maps $X$ to $X'$ and $Y$ to $Y'$.
   A reflection in $z$ maps $X'$ to $X''$ and $Y'$ to $Y''$.
   Prove: $XY \cong X''Y''$

![Reflection Diagram]

**ANIMAL TRACKS** Write a glide reflection that can be used to predict the location of the next track for each set of animal tracks.

31. **turkey**

![Turkey Tracks]

32. **duck**

![Duck Tracks]
33. **KNITTING** Tonisha is knitting a scarf using the tumbling blocks pattern shown at the right. Describe the transformations combined to create the pattern.

Describe the transformations that combined to map each figure.

34. 

35. 

36. **PROOF** Write a two-column proof of Theorem 9.2.
Given: A reflection in line $p$ maps $BC$ to $B'C'$.
A reflection in line $q$ maps $B'C'$ to $B''C''$.
$p \parallel q$, $AD = x$

Prove: 

a. $BB'' \perp p$, $BB'' \perp q$

b. $BB'' = 2x$
37. **PROOF** Write a paragraph proof of Theorem 9.3.

![Diagram](image)

Given: Lines \(l\) and \(m\) intersect at point \(P\). \(A\) is any point not on \(l\) or \(m\).

**Prove:**

a. If you reflect point \(A\) in \(m\), and then reflect its image \(A'\) in \(l\), \(A''\) is the image of \(A\) after a rotation about point \(P\).

b. \(m\angle APA'' = 2(m\angle SPR)\)

38. **FIND THE ERROR** Daniel and Lolita are translating \(\triangle XYZ\) along \(\langle 2, 2 \rangle\) and reflecting it in the line \(y = 2\). Daniel says that the transformation is a glide reflection. Lolita disagrees and says that the transformation is a composition of transformations. Is either of them correct? Explain your reasoning.

![Diagram](image)


40. **CHALLENGE** If \(PQRS\) is translated along \(\langle 3, -2 \rangle\), reflected in \(y = -1\), and rotated 90° about the origin, what are the coordinates of \(P''Q''R''S''\)?

![Diagram](image)

41. **REASONING** If an image is to be reflected in the line \(y = x\) and the \(x\)-axis, does the order of the reflections affect the final image? Explain.
42. **OPEN ENDED** Write a glide reflection or composition of transformations that can be used to transform ΔABC to ΔDEF.

43. **REASONING** When two rotations are performed on a single image, does the order of the rotations *sometimes, always, or never* affect the location of the final image? Explain.

44. **WRITING IN MATH** Compare and contrast glide reflections and compositions of transformations.

45. ΔABC is translated along the vector (−2, 3) and then reflected in the x-axis. What are the coordinates of A’ after the transformation?

   A (1, −4)
   B (1, 4)
   C (−1, 4)
   D (−1, −4)

46. **SHORT RESPONSE** What are the coordinates of D’ if CD with vertices C(2, 4) and D(8, 7) is translated along (−6, 2) and then reflected over the y-axis?
Name:

47. **ALGEBRA** Write \( \frac{18x^2 - 2}{3x^2 - 5x - 2} \) in simplest terms.

- **F** \( \frac{18}{3x + 1} \)
- **H** \( \frac{2(3x - 1)}{x - 2} \)
- **G** \( \frac{2(3x + 1)}{x - 2} \)
- **J** \( \frac{2(3x - 1)}{x - 2} \)

48. **SAT/ACT** If \( f(x) = x^3 - x^2 - x \), what is the value of \( f(-3) \)?

49. **60°**

50. **120°**

51. **180°**

Graph each figure and its image along the given vector.

52. \( \triangle FGH \) with vertices \( F(1, -4), G(3, -1), \) and \( H(7, -1) \); \( \langle 2, 6 \rangle \)

53. quadrilateral \( ABCD \) with vertices \( A(-2, 7), B(-1, 4), C(2, 3), \) and \( D(2, 7) \); \( \langle -3, -5 \rangle \)

**AVIATION** A jet is flying northwest, and its velocity is represented by \( \langle -450, 450 \rangle \) miles per hour. The wind is from the west, and its velocity is represented by \( \langle 100, 0 \rangle \) miles per hour.
Name:

54. a. Find the resultant vector for the jet in component form.
   b. Find the magnitude of the resultant.
   c. Find the direction of the resultant.

Each figure shows a preimage and its reflected image in some line. Copy each figure and draw the line of reflection.

55.

56.

57.
Extend 9-4 Geometry Lab: Tessellations - Exercises

Determine whether each regular polygon will tessellate in the plane. Write yes or no. Explain.

1. triangle
2. pentagon
3. 16-gon

Determine whether each pattern is a tessellation. Write yes or no. If so, describe it as regular, semi-regular, or neither and uniform or not uniform.

4. [Image]
5. [Image]
6. [Image]

Draw a tessellation using the following shape(s).

7. octagon and square
8. hexagon and triangle
9. right triangle
10. trapezoid and a parallelogram

11. **WRITING IN MATH** There are only three possible regular tessellations. List the three polygons used to create these tessellations, and explain why they are the only ones possible.

12. **MAKE A CONJECTURE** Describe a figure that you think will tessellate in three dimensional space. Explain your reasoning.
9-5 Symmetry - Check Your Understanding

State whether the figure appears to have line symmetry. Write yes or no. If so, copy the figure, draw all lines of symmetry, and state their number.

1. 

2. 

3. 

State whether the figure has rotational symmetry. Write yes or no. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

4. 

5. 

6.
7. **U.S. CAPITOL** Completed in 1863, the dome is one of the most recent additions to the United States Capitol. It is supported by 36 iron ribs and has 108 windows, divided equally among three levels.

   ![Image of the U.S. Capitol Dome](image)

   a. Excluding the spire of the dome, how many horizontal and vertical lines of symmetry does the dome appear to have?
   b. Does the dome have rotational symmetry? If so, state the order and magnitude of symmetry.

8. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

![Image of a cube](image)

**9-5 Symmetry - Practice and Problem Solving**

State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.
FLAGS State whether each flag design appears to have line symmetry. Write yes or no. If so, copy the flag, draw all lines of symmetry, and state their number.
State whether the figure has rotational symmetry. Write yes or no. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

18. 

19. 

20. 

21. 

22. 

23. 

WHEELS State whether each wheel cover appears to have rotational symmetry. Write yes or no. If so, state the order and magnitude of symmetry.

24.
State whether the figure has plane symmetry, axis symmetry, both, or neither.

CONTAINERS Determine the number of horizontal and vertical planes of symmetry for each container shown below.
34. **PHOTOGRAPHY** Symmetry is an important component of photography. Photographers often use reflection in water to create symmetry in photos. The photo at the right is a long exposure shot of the Eiffel tower reflected in a pool.

![Photo of the Eiffel tower reflected in a pool.]

a. Describe the two-dimensional symmetry created by the photo.
b. Is three-dimensional symmetry applicable? Explain your reasoning.

35. **COORDINATE GEOMETRY** Determine whether the figure with the given vertices has *line* symmetry and/or *rotational* symmetry.

\[ A(-4, 0), B(0, 4), C(4, 0), D(0, -4) \]

36. \[ R(-3, 3), S(-3, -3), T(3, 3) \]

37. \[ F(0, -4), G(-3, -2), H(-3, 2), J(0, 4), K(3, 2), L(3, -2) \]

38. \[ W(-2, 3), X(-3, -3), Y(3, -3), Z(2, 3) \]

39. **ALGEBRA** Graph the function and determine whether the graph has *line* and/or *rotational* symmetry. If so, state the order and magnitude of symmetry, and write the equations of any lines of symmetry.

\[ y = x \]

\[ y = x^2 + 1 \]

\[ y = -x^3 \]

40. **CRYSTALLOGRAPHY** Determine whether the crystals below have *plane* symmetry and/or *axis* symmetry. If so, state the magnitude of symmetry.

![Image of crystals.]
Name:

42. cubic

43. rhombohedral

44. orthorhombic

45. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate rotational symmetry in regular polygons.
   
   a. **GEOMETRIC** Draw an equilateral triangle and determine its order of symmetry.
   
   b. **GEOMETRIC** Repeat the process in part a for a square, a regular pentagon, and a regular hexagon.
   
   c. **TABULAR** Tabulate the order of symmetry for each polygon.
   
   d. **VERBAL** Make a conjecture about the values of the order of symmetry for a regular polygon.

46. **FIND THE ERROR** Jaime says that Figure A has only line symmetry, and Jewel says that Figure A has only rotational symmetry. Is either of them correct? Explain your reasoning.

47. **CHALLENGE** A quadrilateral in the coordinate plane has exactly two lines of symmetry, $y = x - 1$ and $y = -x + 2$. Find a set of possible vertices for the figure. Graph the figure and the lines of symmetry.

48. **REASONING** A regular polyhedron has axis symmetry of order 3, but does not have plane symmetry. What is the figure? Explain.

49. **OPEN ENDED** Draw a figure that has line symmetry but not rotational symmetry. Explain.

50. **WRITING IN MATH** Compare and contrast line symmetry and rotational symmetry.
Name:

51. How many lines of symmetry can be drawn on the picture of the Canadian flag below?

A 0  
B 1  
C 2  
D 4

52. **GRIDDED RESPONSE** What is the order of symmetry for the figure below?

53. **ALGEBRA** A computer company ships computers in wooden crates that each weigh 45 pounds when empty. If each computer weighs no more than 13 pounds, which inequality best describes the total weight in pounds $w$ of a crate of computers that contains $c$ computers?

F  $c \leq 13 + 45w$

G  $c \geq 13 + 45w$

H  $w \leq 13c + 45$

J  $w \geq 13c + 45$

54. **SAT/ACT** What is the slope of the line determined by the linear equation $5x - 2y = 10$?

A  $-\frac{5}{2}$  
B  $\frac{5}{2}$  
C  $\frac{2}{5}$  
D  $\frac{5}{2}$

Triangle $JKL$ has vertices $J(1, 5), K(3, 1),$ and $L(5, 7)$. Graph $\triangle JKL$ and its image after the indicated transformation.

55. Translation: along $\langle -7, -1 \rangle$  
Reflection: in $x$-axis

56. Translation: along $\langle 1, 2 \rangle$  
Reflection: in $y$-axis
57. Quadrilateral $QRST$ is shown at the right. What is the image of point $R$ after a rotation $180^\circ$ counter clockwise about the origin?

58. **AMUSEMENT PARKS** From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are $11^\circ$ and $8^\circ$ respectively, how far apart are the merry-go-round and the Ferris wheel?

   Determine whether the dilation from Figure A to Figure B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

59.

60.

61.
9-6 Dilations - Check Your Understanding

Copy the figure and point $M$. Then use a ruler to draw the image of the figure under a
dilation with center $M$ and the scale factor $k$ indicated.

1. $k = \frac{1}{4}

2. $k = 2$

3. Determine whether the dilation from Figure $B$ to $B'$ is an \textit{enlargement} or a reduction. Then find the scale
factor of the dilation and $x$.

4. \textbf{Biology} Under a microscope, a single-celled organism 200 microns in length appears to be 50 millimeters
long. If 1 millimeter = 1000 microns, what magnification setting (scale factor) was used? Explain your
reasoning.

Graph the image of each polygon with the given vertices after a dilation centered at the origin with the
given scale factor.

5. $W(0, 0), X(6, 6), Y(6, 0); k = 1.5$

6. \begin{align*}
Q(-4, 4), R(-4, -4), S(4, -4), T(4, 4); k &= \frac{1}{2} \\
\end{align*}

7. \begin{align*}
A(-1, 4), B(2, 4), C(3, 2), D(-2, 2); k &= 2 \\
\end{align*}
9-6 Dilations - Practice and Problem Solving

Copy the figure and point S. Then use a ruler to draw the image of the figure under a dilation with center S and the scale factor $k$ indicated.

9. $k = \frac{5}{2}$

10. $k = 3$

11. $k = 0.8$

12. $k = \frac{1}{3}$

13. $k = 2.25$

14. $k = \frac{7}{4}$

Determine whether the dilation from figure $W$ to $W'$ is an enlargement or a reduction. Then find the scale factor of the dilation and $x$. 

Name:
INSECTS When viewed under a microscope, each insect has the measurement given on the picture. Given the actual measure of each insect, what magnification was used? Explain your reasoning.

Cat Flea
Actual Length: 2.5 mm
20. Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

21. \( J(-8, 0), K(-4, 4), L(-2, 0); k = 0.5 \)

22. \( S(0, 0), T(-4, 0), V(-8, -8); k = 1.25 \)

23. \( A(9, 9), B(3, 3), C(6, 0); k = \frac{1}{3} \)

24. \( D(4, 4), F(0, 0), G(8, 0); k = 0.75 \)

25. \( M(-2, 0), P(0, 2), Q(2, 0), R(0, -2); k = 2.5 \)

26. \( W(2, 2), X(2, 0), Y(0, 1), Z(1, 2); k = 3 \)

27. **COORDINATE GEOMETRY** Refer to the graph of \( FGHJ. \)

![Graph of FGHJ]

a. Dilate \( FGHJ \) by a scale factor of \( \frac{1}{2} \) centered at the origin, and then reflect the dilated image in the y-axis.

b. Complete the composition of transformations in part a in reverse order.

c. Does the order of the transformations affect the final image?

d. Will the order of a composition of a dilation and a reflection sometimes, always, or never affect the dilated image? Explain your reasoning.

28. **PHOTOGRAPHY AND ART** To make a grid drawing in the style of Chuck Close, students overlay a \( \frac{1}{4} \)-inch grid on a 5-inch by 7-inch high contrast photo, overlay a \( \frac{1}{2} \)-inch grid on a 10-inch by 14-inch piece of drawing paper, and then sketch the image in each square of the photo to the corresponding square on the drawing paper.

a. What is the scale factor of the dilation?

b. To create an image that is 10 times as large as the original, what size grids are needed?

c. What would be the area of a grid drawing of a 5-inch by 7-inch photo that used 2-inch grids?
29. **MEASUREMENT** Determine whether the image shown is a dilation of \(ABCD\). Explain your reasoning.

![Image of a geometric figure]

30. **COORDINATE GEOMETRY** \(WXYZ\) has vertices \(W(6, 2), X(3, 7), Y(-1, 4),\) and \(Z(4, -2)\).
   a. Graph \(WXYZ\) and find the perimeter of the figure. Round to the nearest tenth.
   b. Graph the image of \(WXYZ\) after a dilation of \(\frac{1}{2}\) centered at the origin.
   c. Find the perimeter of the dilated image. Round to the nearest tenth.
   How is the perimeter of the dilated image related to the perimeter of \(WXYZ\)?

31. **CHANGING DIMENSIONS** A three-dimensional figure can also undergo a dilation. Consider the rectangular prism shown.

![Image of a rectangular prism]

a. Find the surface area and volume of the prism.
b. Find the surface area and volume of the prism after a dilation with a scale factor of 2.
c. Find the surface area and volume of the prism after a dilation with a scale factor of \(\frac{1}{2}\).
d. How many times as great is the surface area and volume of the image as the preimage after each dilation?
e. Make a conjecture as to the effect a dilation with a positive scale factor \(r\) would have on the surface area and volume of a prism.
32. **COORDINATE GEOMETRY** Refer to the graph of $\triangle DEF$.
   a. Graph the dilation of $\triangle DEF$ centered at point $D$ with a scale factor of 3.
   b. Describe the dilation as a composition of transformations including a dilation with a scale factor of 3 centered at the origin.
   c. If a figure is dilated by a scale factor of 3 with a center of dilation $(x, y)$, what composition of transformations, including a dilation with a scale factor of 3 centered at the origin, will produce the same.

![Graph of $\triangle DEF$]

33. **HEALTH** Refer to the information at the left about coronary artery dilation.

![Diagram of coronary artery dilations]

   a. A surgeon inflates a balloon catheter in a patient’s coronary artery, dilating the balloon as shown. Find the scale factor of this dilation.
   b. Find the cross-sectional area of the balloon before and after the dilation.

Each figure shows a preimage and its image after a dilation centered at point $P$. Copy each figure, locate point $P$, and estimate the scale factor.

34. 

![Diagram of preimage and image after dilation]

35. 

![Diagram of preimage and image after dilation]
36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate dilations centered at the origin with negative scale factors.
   a. **GEOMETRIC** Draw \( \triangle ABC \) with points \( A(-2, 0), B(2, -4) \), and \( C(4, 2) \). Then draw the image of \( \triangle ABC \) after a dilation centered at the origin with a scale factor of \(-2\).
   b. **GEOMETRIC** Repeat the dilation with scale factors of \(-\frac{1}{2}\) and \(-3\).
   c. **TABULAR** Record the coordinates for each dilation in a table.
   d. **VERBAL** Make a conjecture about the function relationship for a dilation centered at the origin with a negative scale factor.
   e. **ANALYTICAL** Write the function rule for a dilation centered at the origin with a scale factor of \(-k\).
   f. **VERBAL** Describe a dilation centered at the origin with a negative scale factor as a composition of transformations.

37. **CHALLENGE** Find the equation for the dilated image of the line \( y = 4x - 2 \) if the dilation is centered at the origin with a scale factor of \(1.5\).

38. **WRITING IN MATH** Are parallel lines (parallelism) and collinear points (collinearity) preserved under all transformations? Explain.

39. **REASONING** Determine whether invariant points are *sometimes*, *always*, or *never* maintained for the transformations described below. If so, describe the invariant point(s). If not, explain why invariant points are not possible.
   a. a dilation of \( \triangle XYZ \) centered at the origin with a scale factor of 2
   b. a dilation of \( ABCD \) with a scale factor of 1
   c. a rotation of \( \overline{AB} \) \( 74^\circ \) about \( B \)
   d. a reflection of \( \triangle MNP \) in the \( x \)-axis
   e. a translation of \( PQRS \) along \( \langle 7,3 \rangle \)

40. **OPEN ENDED** Graph a triangle on the coordinate plane. Dilate the triangle so that the area of the dilation is four times the area of the original triangle. State the scale factor and center of your dilation.

41. **WRITING IN MATH** List the transformations that result in congruent figures, similar figures, and equal figures. Explain your reasoning.

42. **EXTENDED RESPONSE** Quadrilateral \( PQRS \) was dilated to form quadrilateral \( WXYZ \).

\[ \text{[Diagram of quadrilateral dilations]} \]

- a. Is the dilation from \( PQRS \) to \( WXYZ \) an enlargement or reduction?
- b. Which number best represents the scale factor for this dilation?
43. **ALGEBRA** How many ounces of pure water must a pharmacist add to 50 ounces of a 15% saline solution to make a solution that is 10% saline?
   A 25  
   B 20  
   C 15  
   D 5

44. Tionna wants to replicate a painting in an art museum. The painting is 3 feet wide and 6 feet long. She decides on a dilation reduction factor of 0.25. What size paper should she use?
   F 4 in. × 8 in.  
   G 6 in. × 12 in.  
   H 8 in. × 16 in.  
   J 10 in. × 20 in.

45. **SAT/ACT** For all \( x \), \((x - 7)^2 = ?
   A \( x^2 - 49 \)
   B \( x^2 + 49 \)
   C \( x^2 - 14x - 49 \)
   D \( x^2 - 14x + 49 \)

State whether the figure appears to have line symmetry. Write yes or no. If so, copy the figure, draw all lines of symmetry, and state their number.

46.

47.

48.

Describe the transformations that combined to map each figure.
51. **PAINTING** A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor’s dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?

Find the value of $x$.

52. $58.9 = 2x$

53. $\frac{108.6}{\pi} = x$

54. $228.4 = \pi x$

55. $\frac{336.4}{x} = \pi$
Extend 9-6 Graphing Technology Lab: Transformations Using Matrices - Analyze the Results

1. Graph the triangle from Step 1. Use the result from $[A] + [B]$ to graph a second triangle. What kind of transformation do you observe?

2. Enter $\begin{bmatrix} 2 & 2 & 2 \\ -6 & -6 & -6 \end{bmatrix}$ into matrix $[C]$. Then find $[A] + [C]$. Graph the triangle from Step 1 and the sum of $[A] + [C]$. Describe the type of transformation that occurs.

3. What matrix would you add to $[A]$ in order to get $\begin{bmatrix} 7 & 10 & 5 \\ 5 & 9 & 10 \end{bmatrix}$?

4. Graph the triangle from Step 1. Use the result from $[B] \cdot [A]$ to graph a second triangle. What kind of transformation do you observe?

5. Enter $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ into matrix $[B]$. Then find $[B] \cdot [A]$. Graph the triangle from Step 1 and the product of $[B] \cdot [A]$. Describe the type of transformation that occurs.

6. By what matrix would you multiply $[A]$ in order to get $\begin{bmatrix} 1 & 2 & 6 \\ 1 & 5 & 4 \end{bmatrix}$?

7. Graph the triangle from Step 1. Use the result from $[A] \cdot 0.5$ to graph a second triangle. What kind of transformation do you observe?

8. Multiply $[A]$ by 2. Graph the triangle from Step 1 and the product of $[A] \cdot 2$. Describe the type of transformation that occurs.

9. **MAKE A CONJECTURE** What type of matrix operation would you use to perform each transformation?
   a. rotation
   b. translation
   c. dilation
   d. reflection
Study Guide and Review - Vocabulary Check - Chapter 9

Choose the term that best completes each sentence.

1. When a transformation is applied to a figure, and then another transformation is applied to its image, this is a(n) (composition of transformations, order of symmetries).

2. If a figure is folded across a straight line and the halves match exactly, the fold line is called the (line of reflection, line of symmetry).

3. A (dilation, glide reflection) enlarges or reduces a figure proportionally.
4. The number of times a figure maps onto itself as it rotates from 0° to 360° is called the (magnitude of symmetry, order of symmetry).

5. A (line of reflection, translation vector) is the same distance from each point of a figure and its image.

6. A figure has (a center of rotation, symmetry) if it can be mapped onto itself by a rigid motion.

7. A glide reflection includes both a reflection and a (rotation, translation).

8. To rotate a point (90°, 180°) counterclockwise about the origin, multiply the y-coordinate by \(-1\) and then interchange the x- and y-coordinates.

9. A (vector, reflection) is a congruence transformation.

10. A figure has (plane symmetry, rotational symmetry) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

**Study Guide and Review - Lesson-by-Lesson Review - Chapter 9**

Graph each figure and its image under the given reflection.

11. rectangle $ABCD$ with $A(2, -4)$, $B(4, -6)$, $C(7, -3)$, and $D(5, -1)$ in the x-axis.

12. Triangle $XYZ$ with $X(-1, 1)$, $Y(-1, -2)$, and $Z(3, -3)$ in the y-axis.

13. quadrilateral $QRST$ with $Q(-4, -1)$, $R(-1, 2)$, $S(2, 2)$, and $T(0, -4)$ in the line $y = x$.

14. ART Anita is making the two-piece sculpture shown for a memorial garden. In her design, one piece of the sculpture is a reflection of the other, to be placed beside a sidewalk that would be located along the line of reflection. Copy the figures and draw the line of reflection.

15. Graph $\triangle ABC$ with vertices $A(0, -1)$, $B(2, 0)$, $C(3, -3)$ and its image along $(-5, -1)$.

16. Copy the figure and the given translation vector Then draw the translation of the figure along the translation vector.
17. **DANCE** Five dancers are positioned onstage as shown. Dancers $B$, $F$, and $C$ move along $\langle 0, -2 \rangle$ while dancer $A$ moves along $\langle 5, -1 \rangle$. Draw the dancers’ final positions.

18. Copy trapezoid $CDEF$ and point $P$. Then use a protractor and ruler to draw a $50^\circ$ rotation of $CDEF$ about point $P$.

Graph each figure and its image after the specified rotation about the origin.

19. $\triangle MNO$ with vertices $M(-2, 2)$, $N(0, -2)$, $O(1, 0)$; $180^\circ$

20. $\triangle DGF$ with vertices $D(1, 2)$, $G(2, 3)$, $F(1, 3)$; $90^\circ$

Each figure shows a preimage and its image after a rotation about a point $P$. Copy each figure, locate point $P$, and find the angle of rotation.

21.

22.

Graph each figure with the given vertices and its image after the indicated transformation.

23. $\overrightarrow{CD}$: $C(3, 2)$ and $D(1, 4)$
   Reflection: in $y = x$ Rotation: $270^\circ$ about the origin.
24. \( GH : G(-2, -3) \) and \( H(1, 1) \)
   Translation: along \( (4, 2) \) Reflection: in the \( x \)-axis

25. **Patterns** Jeremy is creating a pattern for the border of a poster using a stencil. Describe the transformation combination that he used to create the pattern below.

26. Copy and reflect figure \( T \) in line \( l \) and then line \( m \). Then describe a single transformation that maps \( T \) onto \( T'' \).

   State whether each figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

27.

28.

29. State whether each figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

30.
31. **KNITTING** Amy is creating a pattern for a scarf she is knitting for her friend. How many lines of symmetry are there in the pattern?

32. Copy the figure and point $S$. Then use a ruler to draw the image of the figure under a dilation with center $S$ and scale factor $r = 1.25$.

33. Determine whether the dilation from figure $W$ to $W'$ is an enlargement or a reduction. Then find the scale factor of the dilation and $x$.

34. **CLUBS** The members of the Math Club use an overhead projector to make a poster. If the original image was 6 inches wide, and the image on the poster is 4 feet wide, what is the scale factor of the enlargement?
Chapter 9 - Transformations and Symmetry - Practice Test

1. Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

2. 

3. PROJECTS Eduardo wants to enlarge the picture below to 4 inches by 6 inches for a school project. If his school’s copy machine can only enlarge up to 150% by whole number percents, find two whole number percents by which he can enlarge the piece and get as close to 4 inches by 6 inches or less.

   Copy the figure and point M. Then use a ruler to draw the image of the figure under dilation with center M and the scale factor $r$ indicated.

4. \( r = 1.5 \)

5. \( r = \frac{1}{3} \)
Graph each figure and its image under the given transformation.

9. $\Box FGHIJ$ with vertices $F(-1, 4), G(4, 4), H(3, 1),$ and $J(-2, 1)$ in the x-axis

10. $\triangle ABC$ with vertices $A(0, -1), B(2, 0), C(3, -3); \langle -5, 4 \rangle$

11. quadrilateral $WXYZ$ with vertices $W(2, 3), X(1, 1), Y(3, 0), Z(5, 2); 180^\circ$ about the origin

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

12.

13.
14. **ART** An artist’s rendition of what Stonehenge, a famous archaeological site in England, would have looked like before the stones fell or were removed, is shown below. What is the order and magnitude of symmetry for the outer ring?

![Diagram of Stonehenge](image1)

15. **MULTIPLE CHOICE** What transformation or combination of transformations does the figure below represent?

A dilation  
B glide reflection  
C rotation  
D translation
Chapter 9 - Transformations and Symmetry - Preparing for Standardized Tests - Chapter 9

Solve each problem. Show your work. Responses will be graded using the short-response scoring rubric given at the beginning of the lesson.

1. A flea landed on a coordinate grid. The flea hopped across the $x$-axis and then across the $y$-axis in the form of two consecutive reflections. Then it walked 9 units to the right and 4 units down. If the flea’s final position was at $(4, -1)$, what point did it originally land on?

2. The coordinate grid below shows the final image when a point was rotated $90^\circ$ clockwise about the origin, dilated by a scale factor of 2, and shifted 7 units right. What were the original coordinates?

![Coordinate Grid](https://via.placeholder.com/150)

3. Figure $ABCD$ is an isosceles trapezoid.

![Trapezoid](https://via.placeholder.com/150)

Which of the following are the coordinates of an endpoint of the median of $ABCD$?

- A. $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$
- B. $\left(\frac{2c-a}{2}, \frac{b}{2}\right)$
- C. $\left(\frac{c}{2}, 0\right)$
- D. $\left(\frac{a}{2}, \frac{b}{2}\right)$

4. If the measure of an interior angle of a regular polygon is 108, what type of polygon is it?
- F octagon
- G hexagon
- H pentagon
- J triangle
Chapter 9 - Transformations and Symmetry - Standardized Test Practice - Cumulative, Chapters 1-9

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Point $N$ has coordinates $(4, -3)$. What will the coordinates of its image be after a reflection across the $y$-axis?
   A $N'(−3, 4)$
   B $N'(-4,3)$
   C $N'(4, 3)$
   D $N'(−4, −3)$

2. Which pair of figures shows a reflection across the line followed by a translation up?
   ![Diagrams]
   F

3. What is the angle of rotation that maps point $T$ onto $T'$ in the figure below?
   ![Diagram]
   A $90^\circ$
   B $120^\circ$
   C $135^\circ$
   D $145^\circ$
4. **Given:** \(a \parallel b\)

Which statement below justifies the conclusion that \(\angle 1 \equiv \angle 2\)?

- **F** If \(a \parallel b\) and are cut by transversal \(t\), then alternate exterior angles are congruent.
- **G** If \(a \parallel b\) and are cut by transversal \(t\), then alternate interior angles are congruent.
- **H** If \(a \parallel b\) and are cut by transversal \(t\), then corresponding angles are congruent.
- **J** If \(a \parallel b\) and are cut by transversal \(t\), then vertical angles are congruent.

5. What is the geometric mean of 8 and 18?
   - **A** 9
   - **B** 10
   - **C** 11
   - **D** 12

6. Which of the following is a side length in isosceles triangle \(DEF\)?

   - **F** 2 cm
   - **G** 8 cm
   - **H** 9 cm
   - **J** 11 cm

7. Which of the following has exactly two pairs of consecutive congruent sides?
   - **A** kite
   - **B** parallelogram
   - **C** rhombus
   - **D** trapezoid

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**
8. State whether the figure has rotational symmetry. If so, copy the figure, locate the center, and state the order and magnitude of symmetry.

9. Dilate the figure shown on the coordinate grid by a scale factor of 1.5 centered at the origin.

10. Complete the following statement.
    *According to the Angle Bisector Theorem, if a point is on the bisector of an angle, then it is _______.*

11. Regina left her office downtown and traveled 3 blocks west and 5 blocks north. Write a translation vector to describe her route.

12. What is the interior angle measure of the regular pentagon?
13. **GRIDDED RESPONSE** A group of 75 students were asked what types of movies they like to watch. The results are shown in the Venn diagram.

![Venn Diagram]

How many students said that they like to watch action and drama movies, but not comedy?

**Record your answers on a sheet of paper. Show your work.**

14. Rodrigo is making a scale model.
   a. The actual length of the Golden Gate Bridge is about 9000 feet. If Rodrigo’s model is 45 inches, what is the scale of his model?
   b. How wide will Rodrigo’s model of the bridge be if the actual width is 90 feet?
   c. In Rodrigo’s model, the tower will be 2.5 inches above the roadway. How high above the roadway is the actual tower?