Chapter 4 - Congruent Triangles - Get Ready for Chapter 4

Classify each angle as right, acute, or obtuse.

1. \( m \angle VQS \)
2. \( m \angle TQV \)
3. \( m \angle PQV \)

4. **ORIGAMI** The origami fold involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each angle as *right*, *acute*, or *obtuse*.

   ![Diagram](image)

   **ALGEBRA** Use the figure to find the indicated variable(s). Explain your reasoning.

5. Find \( x \) if \( m \angle 3 = x - 12 \) and \( m \angle 6 = 72 \).

6. If \( m \angle 4 = 2y + 32 \) and \( m \angle 5 = 3y - 3 \), find \( y \).

Find the distance between each pair of points.

7. \( F(3, 6) \), \( G(7, -4) \)

8. \( X(-2, 5) \), \( Y(1, 11) \)

9. \( R(8, 0) \), \( S(-9, 6) \)

10. \( A(14, -3) \), \( B(9, -9) \)

11. **MAPS** Miranda laid a coordinate grid on a map of a state where each 1 unit is equal to 10 miles. If her city is located at \((-8, -12)\) and the state capital is at \((0, 0)\), find the distance from her city to the capital to the nearest tenth of a mile.
4-1 Classifying Triangles - Check Your Understanding

ARCHITECTURE Classify each triangle as acute, equiangular, obtuse, or right.

1. 

2. 

3. 

Classify each triangle as acute, equiangular, obtuse, or right. Explain your reasoning.

4. \( \triangle ABD \)

5. \( \triangle BDC \)

6. \( \triangle ABC \)

Classify each triangle as equilateral, isosceles, or scalene.
8. If point $K$ is the midpoint of $FH$, classify each triangle in the figure at the right as *equilateral*, *isosceles*, or *scalene*.

9. $\triangle FGH$

10. $\triangle GJL$

11. $\triangle FHL$

**ALGEBRA** Find $x$ and the measures of the unknown sides of each triangle.

12.

13.

14. **JEWELRY** Suppose you are bending stainless steel wire to make the earring shown. The triangular portion of the earring is an isosceles triangle. If 1.5 centimeters are needed to make the hook portion of the earring, how many earrings can be made from 45 centimeters of wire? Explain your reasoning.
4-1 Classifying Triangles - Practice and Problem Solving

Classify each triangle as **acute, equiangular, obtuse, or right**.

15. 

16. 

17. 

18. 

19. 

Classify each triangle as **acute, equiangular, obtuse, or right**.

20. 

Diagram of triangle with angles labeled. 

Diagram of quadrilateral with angles labeled.
23. $\triangle ADB$

24. $\triangle UXZ$

25. $\triangle UWZ$

26. $\triangle UXY$

Classify each triangle as equilateral, isosceles, or scalene.

27. 

28. 

29. 

If point $C$ is the midpoint of $BD$ and point $E$ is the midpoint of $DF$, classify each triangle as equilateral, isosceles, or scalene.

30. $\triangle ABC$

31. $\triangle AEF$

32. $\triangle ADF$

33. $\triangle ACD$

34. $\triangle AED$
35. \( \triangle ABD \)

36. **ALGEBRA** Find \( x \) and the length of each side if \( \triangle ABC \) is an isosceles triangle with \( AB \cong BC \).

37. **ALGEBRA** Find \( x \) and the length of each side if \( \triangle FGH \) is an equilateral triangle.

38. **GRAPHIC ART** Classify each numbered triangle in Kat by its angles and by its sides. Use the corner of a sheet of notebook paper to classify angle measures and a ruler to measure sides.

39. **KALEIDOSCOPE** Josh is building a kaleidoscope using PVC pipe, cardboard, bits of colored paper, and a 12-inch square mirror tile. The mirror tile is to be cut into strips and arranged to form an open prism with a base like that of an equilateral triangle. Make a sketch of the prism, giving its dimensions. Explain your reasoning.

Classify each triangle in the figure by its angles and sides.

40. \( \triangle ABE \)
41. $\triangle EBC$

42. $\triangle BDC$

COORDINATE GEOMETRY Find the measures of the sides of $\triangle XYZ$ and classify each triangle by its sides.

43. $X(-5, 9), Y(2, 1), Z(-8, 3)$

44. $X(7, 6), Y(5, 1), Z(9, 1)$

45. $X(3, -2), Y(1, -4), Z(3, -4)$

46. $X(-4, -2), Y(-3, 7), Z(4, -2)$

47. **PROOF** Write a paragraph proof to prove that $\triangle DBC$ is an acute triangle if $m\angle ADC = 120$ and $\triangle ABC$ is acute.

\[
\begin{array}{c}
A \\
\hspace{2cm} 120^\circ \\
\hspace{2cm} B \\
\hspace{2cm} C
\end{array}
\]

48. **PROOF** Write a two-column proof to prove that $\triangle BCD$ is equiangular if $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$.

\[
\begin{array}{c}
A \\
\hspace{2cm} 2 \\
\hspace{2cm} B \\
\hspace{2cm} 3 \\
\hspace{2cm} D \\
\hspace{2cm} E
\end{array}
\]

ALGEBRA For each triangle, find $x$ and the measure of each side.

49. $\triangle FGH$ is an equilateral triangle with $FG = 3x - 10$, $GH = 2x + 5$, and $HF = x + 20$.

50. $\triangle JKL$ is isosceles with $\overline{JK} \equiv \overline{KL}$, $JK = 4x - 1$, $KL = 2x + 5$, and $LJ = 2x - 1$.

51. $\triangle MNP$ is isosceles with $\overline{MN} \equiv \overline{NP}$. $MN$ is two less than five times $x$, $NP$ is seven more than two times $x$, and $PM$ is two more than three times $x$.

52. $\triangle RST$ is equilateral. $RS$ is three more than four times $x$, $ST$ is seven more than two times $x$, and $TR$ is one more than five times $x$.

53. **CONSTRUCTION** Construct an equilateral triangle. Verify your construction using measurement and justify it using mathematics. *(Hint: Use the construction for copying a segment.)*
54. **STOCKS** Technical analysts use charts to identify patterns that can suggest future activity in stock prices. Triangle charts are most useful when the fluctuation in the price of a stock is decreasing over time.

a. Classify by its sides and angles the triangle formed if a vertical line is drawn at any point on the graph.

b. How would the price have to fluctuate in order for the data to form an obtuse triangle? Draw an example to support your reasoning.

![Stocks Chart](image1)

55. **MULTIPLE REPRESENTATIONS** In the diagram, the vertex opposite side $BC$ is $\angle A$.

a. **GEOMETRIC** Draw four isosceles triangles, including one acute, one right, and one obtuse isosceles triangle. Label the vertices opposite the congruent sides as $A$ and $C$. Label the remaining vertex $B$. Then measure the angles of each triangle and label each angle with its measure.

b. **TABULAR** Measure all the angles of each triangle. Organize the measures for each triangle into a table. Include a column in your table to record the sum of these measures.

c. **VERBAL** Make a conjecture about the measures of the angles that are opposite the congruent sides of an isosceles triangle. Then make a conjecture about the sum of the measures of the angles of an isosceles triangle.

d. **ALGEBRAIC** If $x$ is the measure of one of the angles opposite one of the congruent sides in an isosceles triangle, write expressions for the measures of each of the other two angles in the triangle. Explain.

![Diagram](image2)

56. **FIND THE ERROR** Elaina says that $\triangle DFG$ is obtuse. Ines disagrees, explaining that the triangle has more acute angles than obtuse angles so it must be acute. Is either of them correct? Explain your reasoning.

![Diagram](image3)

57. Equiangular triangles are also right triangles.

58. Equilateral triangles are isosceles.

59. Right triangles are equilateral.

60. **CHALLENGE** An equilateral triangle has sides that measure $5x + 3$ units and $7x - 5$ units. What is the perimeter of the triangle? Explain.
Name:

OPEN ENDED Draw an example of each type of triangle below using a protractor and a ruler. Label the sides and angles of each triangle with their measures. If not possible, explain why not.

61. scalene right

62. isosceles obtuse

63. equilateral obtuse

64. WRITING IN MATH Explain why classifying an equiangular triangle as an acute equiangular triangle is unnecessary.

65. Which type of triangle can serve as a counterexample to the conjecture below?

   If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

A equilateral
B obtuse
C right
D scalene

66. ALGEBRA A baseball glove originally cost $84.50. Kenji bought it at 40% off. How much was deducted from the original price?
   F $50.70
   G $44.50
   H $33.80
   J $32.62

67. GRIDDED RESPONSE Jorge is training for a 20-mile race. Jorge runs 7 miles on Monday, Tuesday, and Friday, and 12 miles on Wednesday and Saturday. After 6 weeks of training, Jorge will have run the equivalent of how many races?

68. SAT/ACT What is the slope of the line determined by the equation $2x + y = 5$?
   A $2$
   B $\frac{1}{2}$
   C $-1$
   D $-2$

Find the distance between each pair of parallel lines with the given equations.

69. $x = -2$
   
   $x = 5$

70. $y = -6$
   
   $y = 1$

71. $y = 2x + 3$
   
   $y = 2x - 7$
Name:

72. \( y = x + 2 \)
    \( y = x - 4 \)

73. **FOOTBALL** When striping the practice football field, Mr. Hawkins first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel?

    **Identify the hypothesis and conclusion of each conditional statement.**

74. If three points lie on a line, then they are collinear.

75. If you are a teenager, then you are at least 13 years old.

76. If \( 2x + 6 = 10 \), then \( x = 2 \).

77. If you have a driver’s license, then you are at least 16 years old.

Refer to the figure at the right.

78. How many planes appear in this figure?

79. Name the intersection of plane \( AEB \) with plane \( N \).

80. Name three points that are collinear.

81. Are points \( D, E, C, \) and \( B \) coplanar?

Identify each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

82. \( \angle 5 \) and \( \angle 3 \)

83. \( \angle 9 \) and \( \angle 4 \)

84. \( \angle 11 \) and \( \angle 13 \)

85. \( \angle 1 \) and \( \angle 11 \)
Explore 4-2 Geometry Lab: Angles of Triangles - Model and Analyze

Analyze the Results

1. Angles $A$, $B$, and $C$ are called interior angles of triangle $ABC$. What type of figure do these three angles form when joined together in Step 3?

2. Make a conjecture about the sum of the measures of the interior angles of a triangle.

Model and Analyze the Results

3. The angle adjacent to $\angle C$ is called an exterior angle of triangle $ABC$. Make a conjecture about the relationship among $\angle A$, $\angle B$, and the exterior angle at $C$.

4. Repeat the steps in Activity 2 for the exterior angles of $\angle A$ and $\angle B$ in each triangle.

5. Make a conjecture about the measure of an exterior angle and the sum of the measures of its nonadjacent interior angles.
4-2 Angles of Triangles - Check Your Understanding

Find the measures of each numbered angle.

1. \[
\angle
\]

2. Find each measure.

3. \[
m \angle 2
\]
DECK CHAIRS The brace of this deck chair forms a triangle with the rest of the chair’s frame as shown. If $m \angle 1 = 102^\circ$ and $m \angle 3 = 53^\circ$, find each measure.

5. $m \angle 4$

6. $m \angle 6$

7. $m \angle 2$

8. $m \angle 5$

9. $m \angle 1$

10. $m \angle 3$

11. $m \angle 2$
Find the measure of each numbered angle.
13. 

14. 

15. 

16. **AIRPLANES** The path of an airplane can be modeled using two sides of a triangle as shown. The distance covered during the plane’s ascent is equal to the distance covered during its descent.

   a. Classify the model using its sides and angles.
   b. The angles of ascent and descent are congruent. Find their measures.

   **Find each measure.**

17. \( m \angle 1 \)

18. \( m \angle 3 \)
Name:

19. \( m \angle 2 \)

20. \( m \angle 4 \)

21. \( m \angle ABC \)

22. \( m \angle JKL \)

23. **WHEELCHAIR RAMP** Suppose the wheelchair ramp shown makes a 12° angle with the ground. What is the measure of the angle the ramp makes with the van door?

24. \( m \angle 1 \)

25. \( m \angle 2 \)

26. \( m \angle 3 \)
27. \( m \angle 4 \)

28. \( m \angle 5 \)

29. \( m \angle 6 \)

ALGEBRA Find the value of \( x \). Then find the measure of each angle.

30. 

31. 

32. 

33. **GARDENING** A landscaper is forming an isosceles triangle in a flowerbed using chrysanthemums. She wants \( m \angle A \) to be three times the measure of \( \angle B \) and \( \angle C \). What should the measure of each angle be?

PROOF Write the specified type of proof.

34. flow proof of Corollary 4.1

35. paragraph proof of Corollary 4.2

Find the measure of each numbered angle.

36. 

37. 

![Diagram of a triangle with angles labeled]

38. **ALGEBRA** Classify the triangle shown by its angles. Explain your reasoning.

39. **ALGEBRA** The measure of the larger acute angle in a right triangle is two degrees less than three times the measure of the smaller acute angle. Find the measure of each angle.

40. Determine whether the following statement is true or false. If false, give a counterexample. If true, give an argument to support your conclusion.
   
   *If the sum of two acute angles of a triangle is greater than 90, then the triangle is acute.*

41. **ALGEBRA** In \( \triangle XYZ \), \( m \angle X = 157 \), \( m \angle Y = y \), and \( m \angle Z = z \). Write an inequality to describe the possible measures of \( \angle Z \). Explain your reasoning.

![Car image with angles labeled]

42. **CARS** Refer to the photo at the right.
   
   a. Find \( m \angle 1 \) and \( m \angle 2 \).
   
   b. If the support for the hood were shorter than the one shown, how would \( m \angle 1 \) change? Explain.
   
   c. If the support for the hood were shorter than the one shown, how would \( m \angle 2 \) change? Explain.

43. **PROOF** Write the specified type of proof.

   **Given:** \( RSTUV \) is a pentagon.

   **Prove:** \( m \angle S + m \angle STU + m \angle TUV + m \angle V + m \angle VRS = 540 \)

![Diagram of a pentagon with angles labeled]
44. flow proof
   Given: $\angle 3 \cong \angle 5$
   Prove: $m\angle 1 + m\angle 2 = m\angle 6 + m\angle 7$

45. MULTIPLE REPRESENTATIONS In this problem, you will explore the sum of the measures of the exterior angles of a triangle.
   a. GEOMETRIC Draw five different triangles, extending the sides and labeling the angles as shown. Be sure to include at least one obtuse, one right, and one acute triangle.
   b. TABULAR Measure the exterior angles of each triangle. Record the measures for each triangle and the sum of these measures in a table.
   c. VERBAL Make a conjecture about the sum of the exterior angles of a triangle. State your conjecture using words.
   d. ALGEBRAIC State the conjecture you wrote in part c algebraically.
   e. ANALYTICAL Write a paragraph proof of your conjecture.

46. FIND THE ERROR Curtis measured and labeled the angles of the triangle as shown. Arnoldo says that at least one of his measures is incorrect. Explain in at least two different ways how Arnoldo knows that this is true.

47. WRITING IN MATH Explain how you would find the missing measures in the figure shown.

48. OPEN ENDED Construct a right triangle and measure one of the acute angles. Find the measure of the second acute angle using calculation and explain your method. Confirm your result using a protractor.

49. CHALLENGE Find the values of $y$ and $z$ in the figure at the right.
50. **REASONING** If an exterior angle adjacent to $\angle A$ is acute, is $\triangle ABC$ acute, right, obtuse, or can its classification not be determined? Explain your reasoning.

51. **WRITING IN MATH** Explain why a triangle cannot have an obtuse, acute, and a right exterior angle.

52. **PROBABILITY** Mr. Glover owns a video store and wants to survey his customers to find what type of movies he should buy. Which of the following options would be the best way for Mr. Glover to get accurate survey results?
   A surveying customers who come in from 9 p.m. until 10 p.m.
   B surveying customers who come in on the weekend
   C surveying the male customers
   D surveying at different times of the week and day

53. **SHORT RESPONSE** Two angles of a triangle have measures of $35^\circ$ and $80^\circ$. Describe the possible values of the exterior angle measures of the triangle.

54. **ALGEBRA** Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?
   F $2x - 6 = 8$
   G $22x - 6 = 8x$
   H $-8x - 6 = 8x$
   J $22x + 6 = 8x$

55. **SAT/ACT** Joey has 4 more video games than Solana and half as many as Melissa. If together they have 24 video games, how many does Melissa have?
   A 7
   B 9
   C 12
   D 14

   Classify each triangle as acute, equiangular, obtuse, or right.

56.

57.

58.

**COORDINATE GEOMETRY** Find the distance from $P$ to $\ell$.

59. Line $\ell$ contains points $(0, -2)$ and $(1, 3)$. Point $P$ has coordinates $(-4, 4)$.

60. Line $\ell$ contains points $(-3, 0)$ and $(3, 0)$. Point $P$ has coordinates $(4, 3)$. 
Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

61. 

62. 

State the property that justifies each statement.

63. If \( \frac{x}{2} = 7 \), then \( x = 14 \).

64. If \( x = 5 \) and \( b = 5 \), then \( x = b \).

65. If \( XY - AB = WZ - AB \), then \( XY = WZ \).

66. If \( m \angle A = m \angle B \) and \( m \angle B = m \angle C \), then \( m \angle A = m \angle C \).

67. If \( m \angle 1 + m \angle 2 = 90 \) and \( m \angle 2 = m \angle 3 \), then \( m \angle 1 + m \angle 3 = 90 \).
4-3 Congruent Triangles - Check Your Understanding

Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1. 

2. 

3. **TOOLS** Sareeta is changing the tire on her bike and the nut securing the tire looks like the one shown. Which of the sockets below should she use with her wrench to remove the tire? Explain your reasoning.

   ![Socket Options](image)

   In the figure, \(\triangle LMN \cong \triangle QRS\).

4. Find \(x\).

5. Find \(y\).

   Find \(x\). Explain your reasoning.

6. 

   ![Diagram](image)
7.

8. PROOF Write a paragraph proof.
   Given: \( \angle WXZ \cong \angle YXZ, \angle XZW \cong \angle XZY, \)
   \( WX \cong YX, WZ \cong YZ \)
   Prove: \( \triangle WXZ \cong \triangle YXZ \)
4-3 Congruent Triangles - Practice and Problem Solving

Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

9.

10.

11.

12.

Polygon $BCDE \cong$ polygon $RSTU$. Find each value.

13. $x$

14. $y$

15. $z$

16. $w$
Name:

17. **SAILING** To ensure that sailboat races are fair, the boats and their sails are required to be the same size and shape.
   a. Write a congruence statement relating the triangles in the photo.
   b. Name six pairs of congruent segments.
   c. Name six pairs of congruent angles.

![Image of sailboats]

Find $x$ and $y$.

18. 

![Image of triangle with angles]

19. 

![Image of triangle with angles]

20. 

![Image of triangle with angles]

21. **PROOF** Write a two-column proof of Theorem 4.3.

22. **PROOF** Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.
   Congruence of triangles is symmetric. (Theorem 4.4)

Given: $\triangle RST \cong \triangle XYZ$

Prove: $\triangle XYZ \cong \triangle RST$

<table>
<thead>
<tr>
<th>$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $XY \cong RS$, $YZ \cong ST$, $XZ \cong RT$</th>
<th>$\angle R \cong \angle X$, $\angle L \cong \angle S$, $\angle Z \cong \angle T$, $RS \cong XY$, $ST \cong YZ$, $RT \cong XZ$</th>
<th>$\triangle RST \cong \triangle XYZ$</th>
<th>$\triangle XYZ \cong \triangle RST$</th>
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</table>

**PROOF** Write a two-column proof.
Name:

23. **Given:** $\overline{BD}$ bisects $\angle B$. 

   $\overline{BD} \perp \overline{AC}$

**Prove:** $\angle A \cong \angle C$

![Diagram](image)

24. **Given:** $\angle P \cong \angle T$, $\angle S \cong \angle Q$

   $\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$,

   $\overline{RT} \cong \overline{RS}$

   $\overline{PQ} \cong \overline{TS}$

**Prove:** $\triangle PRQ \cong \triangle TRS$

25. **SCRAPBOOKING** Lanie is using a flower-shaped corner decoration punch for a scrapbook she is working on. If she punches the corners of two pages as shown, what property guarantees that the punched designs are congruent? Explain.

![Image](image)

**PROOF** Write the specified type of proof of the indicated part of Theorem 4.4.

26. Congruence of triangles is transitive. (paragraph proof)

27. Congruence of triangles is reflexive. (flow proof)

**ALGEBRA** Draw and label a figure to represent the congruent triangles. Then find $x$ and $y$.

28. $\triangle ABC \cong \triangle DEF$, $AB = 7$, $BC = 9$, $AC = 11 + x$, $DF = 3x - 13$, and $DE = 2y - 5$

29. $\triangle JMN \cong \triangle RST$, $m \angle L = 49$, $m \angle M = 10y$, $m \angle S = 70$, and $m \angle T = 4x + 9$

30. $\triangle JKL \cong \triangle MNP$, $JK = 12$, $LJ = 5$, $PM = 2x - 3$, $m\angle L = 67$, $m\angle K = y + 4$ and $m\angle N = 2y - 15$
31. **PENNANTS** Scott is in charge of roping off an area of 100 square feet for the band to use during a pep rally. He is using a string of pennants that are congruent isosceles triangles.

   ![Image of pennants](image)

   a. List seven pairs of congruent segments in the photo.
   b. If the area he ropes off for the band is a square, how long will the pennant string need to be?
   c. How many pennants will be on the string?

32. **ARCHITECTURE** In the photo of the Chrysler Building at the left, \( \overline{TS} \cong \overline{ZY}, \overline{XY} \cong \overline{RS}, \overline{TS} \cong \overline{ZX}, \angle X \cong \angle R, \angle T \cong \angle Z, \angle Y \cong \angle S, \) and \( \triangle HGF \cong \triangle LKJ. \)

   ![Image of Chrysler Building](image)

   a. Which triangle, if any, is congruent to \( \triangle XYZ \)? Explain your reasoning.
   b. Which side(s) are congruent to \( \overline{JI} \)? Explain your reasoning.
   c. Which angle(s) are congruent to \( \angle G \)? Explain your reasoning.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the following statement.

   *The areas of congruent triangles are equal.*

   a. **VERBAL** Write a conditional statement to represent the relationship between the areas of a pair of congruent triangles.
   b. **VERBAL** Write the converse of your conditional statement. Is the converse true or false? Explain your reasoning.
   c. **GEOMETRIC** If possible, draw two equilateral triangles that have the same area but are not congruent. If not possible, explain why not.
   d. **GEOMETRIC** If possible, draw two rectangles that have the same area but are not congruent. If not possible, explain why not.
   e. **GEOMETRIC** If possible, draw two squares that have the same area but are not congruent. If not possible, explain why not.
   f. **VERBAL** For which polygons will the following conditional and its converse both be true? Explain your reasoning.

   *If a pair of _________ are congruent, then they have the same area.*
34. **PATTERNS** The pattern shown is created using regular polygons.
   a. What two polygons are used to create the pattern?
   b. Name a pair of congruent triangles.
   c. Name a pair of corresponding angles.
   d. If $CB = 2$ inches, what is $AE$? Explain.
   e. What is the measure of $\angle D$? Explain.

35. **FITNESS** A fitness instructor is starting a new aerobics class using fitness hoops. She wants to confirm that all of the hoops are the same size. What measure(s) can she use to prove that all of the hoops are congruent? Explain your reasoning.

36. **WRITING IN MATH** Explain why the order of the vertices is important when naming congruent triangles. Give an example to support your answer.

37. **FIND THE ERROR** Jasmine and Will are evaluating the congruent figures below. Jasmine says that $\triangle CAB \cong \triangle ZYX$ and Will says that $\triangle ABC \cong \triangle YXZ$. Is either of them correct? Explain.

38. **WRITE A QUESTION** A classmate is using the Third Angles Theorem to show that if 2 corresponding pairs of the angles of two triangles are congruent, then the third pair is also congruent. Write a question to help him decide if he can use the same strategy for quadrilaterals.

39. **CHALLENGE** Find $x$ and $y$ if $\triangle PQR \cong \triangle RQS$.
40. Two triangles with two pairs of congruent corresponding angles and three pairs of congruent corresponding sides are congruent.

41. Two triangles with three pairs of corresponding congruent angles are congruent.

42. **CHALLENGE** Write a paragraph proof to prove polygon $ABED \equiv$ polygon $FEBD$.

43. **WRITING IN MATH** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

   $\text{Equilateral triangles are congruent.}$

44. Barrington cut four congruent triangles off the corners of a rectangle to make an octagon as shown below. What is the area of the octagon?

   A 456 cm$^2$
   B 528 cm$^2$
   C 552 cm$^2$
   D 564 cm$^2$

45. **GRIDDED RESPONSE** Triangle $ABC$ is congruent to $\triangle HJI$. The vertices of $\triangle ABC$ are $A(-1, 2)$, $B(0, 3)$ and $C(2, -2)$. What is the measure of side $HI$?

46. **ALGEBRA** Which is a factor of $x^2 + 19x - 42$?
   
   F $x + 14$
   G $x + 2$
   H $x - 2$
   J $x - 14$

47. **SAT/ACT** Mitsu travels a certain distance at 30 miles per hour and returns the same route at 65 miles per hour. What is his average speed in miles per hour for the round trip?
   
   A 35.0
   B 41.0
   C 47.5
   D 55.3
Find each measure.

48. \( m \angle 2 \)

49. \( m \angle 1 \)

50. \( m \angle 3 \)

COORDINATE GEOMETRY Find the measures of the sides of \( \triangle JKL \) and classify each triangle by the measures of its sides.

51. \( J(-7, 10), K(15, 0), L(-2, -1) \)

52. \( J(9, 9), K(12, 14), L(14, 6) \)

53. \( J(4, 6), K(4, 11), L(9, 6) \)

54. \( J(16, 14), K(7, 6), L(-5, -14) \)

55. Two angles that form a linear pair are supplementary.

56. If two angles are supplementary, then one of the angles is obtuse.

57. CARPENTRY A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a 90° corner results?

58. Copy and complete the proof.

Given: \( MN \cong PQ, PQ \cong RS \)

Prove: \( MN \cong RS \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ?</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( MN = PQ, PQ = RS )</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. ?</td>
<td>c. ?</td>
</tr>
<tr>
<td>d. ( MN \cong RS )</td>
<td>d. Definition of congruent segments</td>
</tr>
</tbody>
</table>
4-4 Proving Triangles Congruent-SSS, SAS - Check Your Understanding

1. **OPTICAL ILLUSION** $ABCD$ is a square with $AB = CD$ and $DA = BC$. $ABCD$ is congruent to the three other squares that form the pattern.
   a. How many different-sized triangles are used to create the illusion?
   b. Use the Side-Side-Side Congruence Postulate to prove that $\triangle ABC \cong \triangle CDA$.
   c. What is the relationship between the lines formed by the bases of the triangles? Explain your reasoning.

![Diagram of a square with triangles]

2. **EXTENDED RESPONSE** Triangle $ABC$ has vertices $A(-3, -5)$, $B(-1, -1)$, and $C(-1, -5)$. Triangle $XYZ$ has vertices $X(5, -5)$, $Y(3, -1)$, and $Z(3, -5)$.
   a. Graph both triangles on the same coordinate plane.
   b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.
   c. Write a logical argument using coordinate geometry to support your conjecture.

![Diagram of triangles]

3. **EXERCISE** In the exercise diagram, if $\overline{LP} \cong \overline{NO}$, $\angle LPM \cong \angle NOM$, and $\triangle MOP$ is equilateral, write a paragraph proof to show that $\triangle LMP \cong \triangle NMO$.

![Exercise diagram]

Write a two-column proof.

4. **Given:** $\overline{BA} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$
   **Prove:** $\overline{BC} \cong \overline{DA}$

![Diagram to prove congruence]
4-4 Proving Triangles Congruent-SSS, SAS - Practice and Problem Solving

PROOF Write the specified type of proof.
5. paragraph proof
Given: \( QR \cong SR, \quad ST \cong QT \)
Prove: \( \triangle QRT \cong \triangle SRT \)

6. two-column proof
Given: \( AB \cong ED, \, CA \cong CE, \quad AC \) bisects \( BD \).
Prove: \( \triangle ABC \cong \triangle EDC \)

7. BRIDGES The Sunshine Skyway Bridge in Florida is the world’s longest cable-stayed bridge, spanning 4.1 miles of Tampa Bay. It is supported using steel cables suspended from two concrete supports. If the supports are the same height above the roadway and perpendicular to the roadway, and the topmost cables meet at a point midway between the supports, prove that the two triangles shown in the photo are congruent.

Determine whether \( \triangle MNO \cong \triangle QRS \). Explain.

8. \( M(2, 5), \, N(5, 2), \, O(1, 1), \, Q(-4, 4), \, R(-7, 1), \, S(-3, 0) \)

9. \( M(0, -1), \, N(-1, -4), \, O(-4, -3), \, Q(3, -3), \, R(4, -4), \, S(3, 3) \)

10. \( M(0, -3), \, N(1, 4), \, O(3, 1), \, Q(4, -1), \, R(6, 1), \, S(9, -1) \)

11. \( M(4, 7), \, N(5, 4), \, O(2, 3), \, Q(2, 5), \, R(3, 2), \, S(0, 1) \)

PROOF Write the specified type of proof.
12. two-column proof
   Given: $BD \perp AC$,
   \[ BD \text{ bisects } AC. \]
   Prove: $\triangle ABD \cong \triangle CBD$

[Diagram of a triangle with vertices A, B, C, D, and acute angles at B and D]

13. paragraph proof
   Given: $R$ is the midpoint of $QS$ and $PT$.
   Prove: $\triangle PRQ \cong \triangle TRS$

   PROOF Write the specified type of proof.

[Diagram of two triangles sharing a side QR]

14. flow proof
   Given: $JM \equiv NK$; $L$ is the midpoint of $JN$ and $KM$.
   Prove: $\angle M J L \equiv \angle K N L$

[Diagram of a quadrilateral with a line segment JK and point L at the midpoint of JN]

15. paragraph proof
   Given: $\triangle XYZ$ is equilateral. $\overline{WY}$ bisects $\angle Y$.
   Proof: $\overline{XW} \equiv \overline{ZW}$

[Diagram of an equilateral triangle with a line segment WY bisecting angle Y]

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write not possible.

16. 

[Diagram of two overlapping triangles]
17.

18.

19.

20. SIGNs Refer to the diagram at the right.
   a. Identify the three-dimensional figure represented by the wet floor sign.
   b. If $AB \equiv AD$ and $CB \equiv DC$, prove that $\triangle ACB \equiv \triangle ACD$.
   c. Why do the triangles not look congruent in the diagram?

PROOF Write a flow proof.

21. Given: $\overline{MJ} \equiv \overline{ML}$, $K$ is the midpoint of $\overline{JL}$.
    Prove: $\triangle MJK \equiv \triangle MLK$

22. Given: $\triangle TPQ \equiv \triangle SPR$
    $\angle TQR \equiv \angle SRQ$
    Prove: $\triangle TQR \equiv \triangle SRQ$
23. **SOFTBALL** Use the diagram of a fast-pitch softball diamond shown.
   a. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.
   b. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.

![Diagram of a softball diamond]

**PROOF** Write a two-column proof.

24. **Given:** \(YX \cong WZ, YX \parallel ZW\)
    **Prove:** \(\triangle YXZ \cong \triangle WZX\)

![Diagram of a triangle]

25. **Given:** \(\triangle EAB \cong \triangle DCE\)
    **Prove:** \(\triangle EAD \cong \triangle DCE\)

![Diagram of a triangle]

26. **Write a paragraph proof.**
    **Given:** \(HI \cong HM, PM \cong KL, PG \cong KJ, GH \cong JH\)
    **Prove:** \(\angle G \cong \angle J\)

![Diagram of a triangle]

**ALGEBRA** Find the value of the variable that yields congruent triangles. Explain.

27. \(\triangle WXY \cong \triangle WXZ\)

![Diagram of a triangle]
28. \( \triangle ABC \cong \triangle FGH \)

29. **CHALLENGE** Refer to the graph shown.
   a. Describe two methods you could use to prove that \( \triangle WYZ \) is congruent to \( \triangle WYX \). You may not use a ruler or a protractor. Which method do you think is more efficient? Explain.
   b. Are \( \triangle WYZ \) and \( \triangle WYX \) congruent? Explain your reasoning.

30. **REASONING** Determine whether the following statement is true or false. If true, explain your reasoning. If false, provide a counterexample.
   If the congruent sides in one isosceles triangle have the same measure as the congruent sides in another isosceles triangle, then the triangles are congruent.

31. **FIND THE ERROR** Bonnie says that \( \triangle PRQ \cong \triangle XYZ \) by SAS. Shada disagrees. She says that there is not enough information to prove that the two triangles are congruent. Is either of them correct? Explain.

32. **OPEN ENDED** Use a straightedge to draw obtuse triangle \( \triangle ABC \). Then construct \( \triangle XYZ \) so that it is congruent to \( \triangle ABC \) using either SSS or SAS. Justify your construction mathematically and verify it using measurement.

33. **WRITING IN MATH** Two pairs of corresponding sides of two right triangles are congruent. Are the triangles congruent? Explain your reasoning.

34. **ALGEBRA** The Ross Family drove 300 miles to visit their grandparents. Mrs. Ross drove 70 miles per hour for 65% of the trip and 35 miles per hour or less for 20% of the trip that was left. Assuming that Mrs. Ross never went over 70 miles per hour, how many miles did she travel at a speed between 35 and 70 miles per hour?
   A 195
   B 84
   C 21
   D 18
35. In the figure, $\angle C \cong \angle Z$ and $\overline{AC} \cong \overline{XZ}$.

What additional information could be used to prove that $\triangle ABC \cong \triangle XYZ$?

- F $BC \cong YZ$
- G $AB \cong XY$
- H $BC \cong XZ$
- J $XZ \cong XY$

36. **EXTENDED RESPONSE** The graph below shows the eye colors of all of the students in a class. What is the probability that a student chosen at random from this class will have blue eyes? Explain your reasoning.

![Class Eye Color Chart](chart.png)

37. SAT/ACT If $4a + 6b = 6$ and $-2a + b = -7$, what is the value of $a$?

- A $-1$
- B $2$
- C $3$
- D $4$

In the diagram, $\triangle LMNP \cong \triangle QRST$.

![Diagram](diagram.png)

38. Find $x$.

39. Find $y$.

40. **ASTRONOMY** The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form $\triangle RSA$. If $\angle R$ = 41 and $\angle S$ = 109, find $\angle A$.

Write an equation in slope-intercept form for each line.

41. $(-5, -3)$ and $(10, -6)$

42. $(4, -1)$ and $(-2, -1)$
43. \((-4, -1)\) and \((-8, -5)\)

Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.

44. If \(x^2 = 25\), then \(x = 5\).

45. If you are 16, you are a junior in high school.

State the property that justifies each statement.

46. \(AB = AB\)

47. If \(EF = GH\) and \(GH = JK\), then \(EF = JK\).

48. If \(a^2 = b^2 - c^2\), then \(b^2 - c^2 = a^2\).

49. If \(XY + 20 = YW\) and \(XY + 20 = DT\), then \(YW = DT\).

**Extend 4-4 Geometry Lab: Proving Constructions - Exercises**

1. Construct a line parallel to a given line through a given point. Write a two-column proof of your construction.

2. Construct an equilateral triangle. Write a paragraph proof of your construction.

3. **CHALLENGE** Construct the bisector of a segment that is also perpendicular to the segment and write a two-column proof of your construction. (*Hint: You will need to use more than one pair of congruent triangles.*)
Chapter 4 - Congruent Triangles - Mid-Chapter Quiz: Lessons
4-1 through 4-4

1. **COORDINATE GEOMETRY** Classify \( \Delta ABC \) with vertices \( A(-2, -1), B(-1, 3), \) and \( C(2, 0) \) as scalene, equilateral, or isosceles.

2. **MULTIPLE CHOICE** Which of the following are the measures of the sides of isosceles triangle \( QRS? \)
   
   - A 17, 17, 15
   - B 15, 15, 16
   - C 14, 15, 14
   - D 14, 14, 16

3. **ALGEBRA** Find \( x \) and the length of each side if \( \Delta WXY \) is an equilateral triangle with sides \( \overline{WX} = 6x - 12, \overline{XY} = 2x + 10, \) and \( \overline{WY} = 4x - 1. \)

   Find the measure of each angle indicated.

4. \( m \angle 1 \)

5. \( m \angle 2 \)

6. \( m \angle 3 \)
7. **ASTRONOMY** Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form \( \triangle LEO \). If the angles have measures as shown in the figure, find \( m \angle OLE \).

![Diagram of Leo constellation]

Find the measure of each numbered angle.

8. \( m \angle 4 \)
9. \( m \angle 5 \)
10. \( m \angle 6 \)
11. \( m \angle 7 \)

In the diagram, \( \triangle RST \cong \triangle ABC \).

![Diagram of triangles RST and ABC]

12. Find \( x \).

13. Find \( y \).

14. **ARCHITECTURE** The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent.

![Diagram of an A-frame house]
15. **MULTIPLE CHOICE** Determine which statement is true given that $\triangle CBX \cong \triangle SM$.
   
   F $\overline{MO} \equiv \overline{SL}$  
   G $\overline{XC} \equiv \overline{ML}$  
   H $\angle X \equiv \angle S$  
   J $\angle XCB \equiv \angle LSM$

16. **BRIDGES** A bridge truss is shown in the diagram below, where $\overline{AC} \perp \overline{BD}$ and $B$ is the midpoint of $\overline{AC}$. What method can be used to prove that $\triangle ABD \cong \triangle CBD$?

   ![Bridge Truss Diagram](image)

   Determine whether $\triangle PQR \cong \triangle XYZ$.

17.  $P(3, -5), Q(11, 0), R(1, 6), X(5, 1), Y(13, 6), Z(3, 12)$

18.  $P(-3, -3), Q(-5, 1), R(-2, 6), X(2, -6), Y(3, 3), Z(5, -1)$

19.  $P(8, 1), Q(-7, -15), R(9, -6), X(5, 11), Y(-10, -5), Z(6, 4)$

   **Write a two-column proof.**

20. **Given:** $\triangle LMN$ is isos. with $\overline{LM} \equiv \overline{NM}$, and $\overline{MO}$ bisects $\angle LMN$.
    **Prove:** $\triangle MLO \equiv \triangle MNO$
4-5 Proving Triangles Congruent-ASA, AAS - Check Your Understanding

PROOF Write the specified type of proof.

1. two-column proof
   Given: $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$.
   Prove: $\triangle ABC \cong \triangle DBC$

2. flow proof
   Given: $\overline{JK} \parallel \overline{LM}$, $\overline{JL} \parallel \overline{KM}$
   Prove: $\triangle JML \cong \triangle MJK$

3. paragraph proof
   Given: $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, $\overline{JL}$ bisects $\angle KLM$.
   Prove: $\triangle JKL \cong \triangle JML$

4. two-column proof
   Given: $\overline{GH} \parallel \overline{FJ}$
   $m \angle G = m \angle J = 90$
   Prove: $\triangle HJF \cong \triangle FGH$
5. BRIDGE BUILDING A surveyor needs to find the distance from point $A$ to point $B$ across a canyon. She places a stake at $A$, and a coworker places a stake at $B$ on the other side of the canyon. The surveyor then locates $C$ on the same side of the canyon as $A$ such that $\overline{CA} \perp \overline{AB}$. A fourth stake is placed at $E$, the midpoint of $\overline{CA}$. Finally, a stake is placed at $D$ such that $\overline{CD} \perp \overline{CA}$ and $D$, $E$, and $B$ are sited as lying along the same line.

a. Explain how the surveyor can use the triangles formed to find $AB$.

b. If $AC = 1300$ meters, $DC = 550$ meters, and $DE = 851.5$ meters, what is $AB$? Explain your reasoning.

4-5 Proving Triangles Congruent-ASA, AAS - Practice and Problem Solving

PROOF Write the specified type of proof.
6. **Given:** $\overline{CE}$ bisects $BED$, $\angle BCE$ and $\angle ECD$ are right angles.  
**Prove:** $\triangle ECB \cong \triangle ECD$

![Diagram of triangle ECB and ECD with bisector CE](image)

7. **Given:** $\angle W \cong \angle Y$, $WZ \cong YZ$, $XZ$ bisects $\angle WZY$.  
**Prove:** $\triangle XWZ \cong \triangle XYZ$

![Diagram of triangle XWZ and XYZ with bisector XZ](image)

8. **TOYS** The object of the toy shown is to make the two spheres meet and strike each other repeatedly on one side of the wand and then again on the other side. If $\angle JKL \cong \angle MLK$ and $\angle JLK \cong \angle MKL$, prove that $\overline{JK} \cong \overline{ML}$.

PROOF Write a two-column proof.

![Image of TOYS toy](image)

9. **Given:** $V$ is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.  
**Prove:** $\triangle UVY \cong \triangle XVW$

![Diagram of triangles UVY and XVW with midpoint V](image)

10. **Given:** $\overline{MS} \cong \overline{RQ}, \overline{MS} \parallel \overline{RQ}$  
**Prove:** $\triangle MSP \cong \triangle RQP$

![Diagram of triangles MSP and RQP with parallel lines MS and RQ](image)
11. **PROOF** Write a flow proof.
   \[ \angle A \text{ and } \angle C \text{ are right angles.} \]
   \[ \angle ABE \cong \angle CBD, \ AE \cong CD \]
   \[ \text{Prove: } BE \cong BD \]

12. **PROOF** Write a flow proof.
   \[ \text{Given: } KM \text{ bisects } \angle JML; \ \angle J \equiv \angle L. \]
   \[ \text{Prove: } JM \equiv IM \]

13. **FITNESS** A high school wants to hold a 1500-meter regatta on Lake Powell but is unsure if the lake is long enough. To measure the distance across the lake, the crew members locate the vertices of the triangles below and find the measures of the lengths of \( \triangle HJK \) as shown below.

   a. Explain how the crew team can use the triangles formed to estimate the distance FG across the lake.
   b. Using the measures given, is the lake long enough for the team to use as the location for their regatta? Explain your reasoning.

**ALGEBRA** Find the value of the variable that yields congruent triangles.

14. \[ \triangle BCD \cong \triangle WXY \]

15. \[ \triangle MHJ \cong \triangle PQJ \]
16. THEATER DESIGN The trusses of the roof of the outdoor theater shown below appear to be several different pairs of congruent triangles. Assume that trusses that appear to lie on the same line actually lie on the same line.

\[ \overline{AB} \text{ bisects } \angle CBD \text{ and } \angle CAD, \text{ prove that } \triangle ABC \cong \triangle ABD. \]

\[ \text{b. If } \triangle ABC \cong \triangle ABD \text{ and } \angle FCA \equiv \angle EDA, \text{ prove that } \triangle CAF \cong \triangle DAE. \]

\[ \text{c. If } \overline{HB} \equiv \overline{EB}, \angle BHG \equiv \angle BEA, \angle HJG \equiv \angle EAD, \text{ and } \angle JGB \equiv \angle DAB, \text{ prove that } \triangle BHG \cong \triangle BEA. \]

PROOF Write a paragraph proof.

17. Given: \( \overline{AE} \perp \overline{DE}, \overline{EA} \perp \overline{AB}, \) \( C \) is the midpoint of \( \overline{AE}. \)
Prove: \( CD \equiv CB \)

18. Given: \( \angle F \cong \angle J, \overline{FH} \parallel \overline{GJ} \)
Prove: \( \overline{FH} \equiv \overline{GJ} \)

PROOF Write a two-column proof.

19. Given: \( \angle K \cong \angle M, \overline{KP} \perp \overline{PR}, \overline{MR} \perp \overline{PR} \)
Prove: \( \angle KPL \equiv \angle MRL \)
20. **Given:** $QR \cong SR \cong WR \cong VR$

**Prove:** $QT \cong WU$

---

21. **FITNESS** The seat tube of a bicycle forms a triangle with each seat and chain stay as shown. If each seat stay makes a $44^\circ$ angle with its corresponding chain stay and each chain stay makes a $68^\circ$ angle with the seat tube, show that the two seat stays are the same length.

---

22. **OPEN ENDED** Draw and label two triangles that could be proved congruent by ASA.

---

23. **FIND THE ERROR** Tyrone says it is not possible to show that $\triangle ADE \cong \triangle ACB$. Lorenzo disagrees, explaining that since $\angle ADE \cong \angle ACB$, and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ADE \cong \triangle ACB$. Is either of them correct? Explain.

---

24. **REASONING** Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles.

---

25. **CHALLENGE** Using the information given in the diagram, write a flow proof to show that $\triangle PVQ \cong \triangle SVT$.

---

26. **WRITING IN MATH** Summarize the methods described in Lessons 4-3, 4-4, and 4-5 for proving triangle congruence into a chart that explains when to use each method.
27. Given: \( BC \) is perpendicular to \( AD \); \( \angle 1 \cong \angle 2 \).

Which theorem or postulate could be used to prove \( \triangle ABC \cong \triangle DBC \)?
A AAS
B ASA
C SAS
D SSS

28. SHORT RESPONSE Write an expression that can be used to find the values of \( s(n) \) in the table.

\[
\begin{array}{|c|c|c|c|c|}
\hline
n & -8 & -4 & -1 & 0 & 1 \\
\hline
s(n) & 1.00 & 2.00 & 2.75 & 3.00 & 3.25 \\
\hline
\end{array}
\]

29. ALGEBRA If \(-7\) is multiplied by a number greater than 1, which of the following describes the result?
   F a number greater than 7
   G a number between \(-7\) and 7
   H a number greater than \(-7\)
   J a number less than \(-7\)

30. SAT/ACT \( \sqrt{121+104} = ? \)
   A 15
   B 21
   C 125
   D 225

Determine whether \( \triangle ABC \cong \triangle XYZ \). Explain.

31. \( A(6, 4), B(1, -6), C(-9, 5), X(0, 7), Y(5, -3), Z(15, 8) \)

32. \( A(0, 5), B(0, 0), C(-2, 0), X(4, 8), Y(4, 3), Z(6, 3) \)

33. ALGEBRA If \( \triangle RST \cong \triangle JKL \), \( RS = 7, ST = 5, RT = 9 + x, JL = 2x - 10 \), and \( JK = 4y - 5 \), draw and label a figure to represent the congruent triangles. Then find \( x \) and \( y \).

34. BUSINESS Maxine charges $5 to paint a mailbox and $4 per hour to mow a lawn. Write an equation to represent the amount of money Maxine can earn from a homeowner who has his or her mailbox painted and lawn mowed.

Copy and complete each truth table.

35.

\[
\begin{array}{|c|c|c|c|}
\hline
p & q & \sim p \lor q \\
\hline
F & T & \\
T & T & \\
F & F & \\
T & F & \\
\hline
\end{array}
\]
PROOF Write a two-column proof for each of the following.

37. Given: \( \angle 2 \cong \angle 1 \)
\( \angle 1 \cong \angle 3 \)
Prove: \( AB \parallel DE \)

38. Given: \( \triangle MJK \cong \triangle KLM \)
\( \angle LMJ \) and \( \angle KLM \) are supplementary.
Prove: \( \overline{KJ} \parallel \overline{LM} \)
Extend 4-5 Geometry Lab: Congruence in Right Triangles

- Analyze

Study each pair of right triangles.

Analyze

a.

b.

c.

1. Is each pair of triangles congruent? If so, which congruence theorem or postulate applies?

2. Rewrite the congruence rules from Exercise 1 using leg, (L), or hypotenuse, (H), to replace side. Omit the A for any right angle since we know that all right triangles contain a right angle and all right angles are congruent.

3. **MAKE A CONJECTURE** If you know that the corresponding legs of two right triangles are congruent, what other information do you need to declare the triangles congruent? Explain.

Analyze

4. Does the model yield a unique triangle?

5. Can you use the lengths of the hypotenuse and a leg to show right triangles are congruent?

6. **Make a conjecture** about the case of SSA that exists for right triangles.

Determine whether each pair of triangles is congruent. If yes, tell which postulate or theorem applies.

7.

8.
Name:

**PROOF** Write a proof for each of the following.

10. Theorem 4.6

11. Theorem 4.7

12. Theorem 4.8 (*Hint*: There are two possible cases.)

13. Theorem 4.9

Use the figure at the right.

14. Given: \( \overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC} \)

\[ \overline{AC} \cong \overline{BD} \]

Prove: \( \overline{AB} \cong \overline{DC} \)

15. Given: \( \overline{AB} \parallel \overline{DC}, \overline{AB} \perp \overline{BC} \)

\[ \text{E is the midpoint of } \overline{AC} \text{ and } \overline{BD}. \]

Prove: \( \overline{AC} \cong \overline{DB} \)
4-6 Isosceles and Equilateral Triangles - Check Your Understanding

Refer to the figure at the right.

1. If \( AB \cong CB \), name two congruent angles.

2. If \( \angle EAC \cong \angle ECA \), name two congruent segments.

Find each measure.

3. \( FH \)

4. \( m \angle MRP \)

ALGEBRA Find the value of each variable.

5. 

6. 

---
7. PROOF Write a two-column proof.
   Given: \( \triangle ABC \) is isosceles; \( \overline{EB} \) bisects \( \angle ABC \).
   Prove: \( \triangle ABE \cong \triangle CBE \)

8. ROLLER COASTERS The roller coaster track shown in the photo on page 283 appears to be composed of congruent triangles. A portion of the track is shown.
   a. If \( \overline{QR} \) and \( \overline{ST} \) are perpendicular to \( \overline{QT} \), \( \triangle VSR \) is isosceles with base \( \overline{SR} \), and \( \overline{QT} \parallel \overline{SR} \), prove that \( \triangle RQV \cong \triangle STV \).
   b. If \( VR = 2.5 \) meters and \( QR = 2 \) meters, find the distance between \( \overline{QR} \) and \( \overline{ST} \). Explain your reasoning.
4-6 Isosceles and Equilateral Triangles - Practice and Problem Solving

Refer to the figure at the right.

9. If $AB \cong AE$, name two congruent angles.

10. If $\angle AFB \cong \angle AFB$, name two congruent segments.

11. If $CA \cong DA$, name two congruent angles.

12. If $\angle DAE \cong \angle DEA$, name two congruent segments.

13. If $\angle BCF \cong \angle BFC$, name two congruent segments.

14. If $FA \cong AH$, name two congruent angles.

Find each measure.

15. $m \angle BAC$

16. $m \angle SRT$

17. $TR$

18. $CB$
Name:

ALGEBRA Find the value of each variable.

19. 

![Image 2x11 and 6x-9](image)

20. 

![Image 3x and (3x+6)°](image)

21. 

![Image 52° and 2y+6](image)

22. 

![Image 60° and 5x-3](image)

PROOF Write a paragraph proof.

23. Given: $\triangle HJM$ is an isosceles triangle, and $\triangle HKL$ is an equilateral triangle. $\angle JKH$ and $\angle HKL$ and $\angle HLK$ and $\angle MLH$ are supplementary.  
Prove: $\angle JHK \cong \angle MHL$

![Image](image)

24. Given: $\overline{XY} \cong \overline{XZ}$  
$W$ is the midpoint of $\overline{XY}$.  
$Q$ is the midpoint of $\overline{XZ}$.  
Prove: $WZ \cong QY$

![Image](image)
Name:

25. **BABYSITTING** While babysitting her neighbor’s children, Elisa observes that the supports on either side of a park swing set form two sets of triangles. Using a jump rope to measure, Elisa is able to determine that $AB \equiv AC$, but $BC \not\equiv AB$.
   a. Elisa estimates $m \angle BAC$ to be 50. Based on this estimate, what is $m \angle ABC$? Explain.
   b. If $BE \equiv CD$, show that $\triangle AED$ is isosceles.
   c. If $BC \parallel ED$ and $ED \equiv AD$, show that $\triangle AED$ is equilateral.
   d. If $\triangle JKL$ is isosceles, what is the minimum information needed to prove that $\triangle ABC \cong \triangle JKL$? Explain your reasoning.

![Diagram of a swing set with angles and lines marked.](image)

26. **CHIMNEYS** In the picture, $BD \perp AC$ and $\triangle ABC$ is an isosceles triangle with base $AC$. Show that the chimney of the house, represented by $BD$, bisects the angle formed by the sloped sides of the roof, $\angle ABC$.

![Diagram of a chimney with angles marked.](image)

27. **CONSTRUCTION** Construct three different isosceles right triangles. Explain your method. Then verify your constructions using measurement and mathematics.

28. **PROOF** Based on your construction in Exercise 27, make and prove a conjecture about the relationship between the base angles of an isosceles right triangle.

![Diagram of an isosceles right triangle with angles marked.](image)

**Find each measure.**

29. $m \angle CAD$

30. $m \angle ACD$

31. $m \angle ACB$
32. $m \angle ABC$

33. **FITNESS** In the diagram, the rider will use his bike to hop across the tops of each of the concrete solids shown. If each triangle is isosceles with vertex angles $G$, $H$, and $J$, and $BG \cong HC$, $HD \cong JF$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$, show that the distance from $B$ to $F$ is three times the distance from $D$ to $F$.

![Diagram of concrete solids with a bicycle rider]

34. Given: $\triangle WXY$ is isosceles; $\overline{ZY} \perp \overline{XY}$.
Prove: $\angle X$ and $\angle YZV$ are complementary.

**PROOF** Write a two-column proof of each corollary or theorem.

35. Corollary 4.3

36. Corollary 4.4

37. Theorem 4.11

**Find the value of each variable.**

38. ![Diagram of a triangle with algebraic expressions]

39. ![Diagram of a triangle with algebraic expressions]
GAMES Use the diagram of a game timer shown to find each measure.

40. \( m \angle LPM \)

41. \( m \angle LMP \)

42. \( m \angle JLK \)

43. \( m \angle JKL \)

44. MULTIPLE REPRESENTATIONS In this problem, you will explore possible measures of the interior angles of an isosceles triangle given the measure of one exterior angle.
   a. GEOMETRIC Use a ruler and a protractor to draw three different isosceles triangles, extending one of the sides adjacent to the vertex angle and to one of the base angles, and labeling as shown.
   b. TABULAR Use a protractor to measure and record \( m \angle 1 \) for each triangle. Use \( m \angle 1 \) to calculate the measures of \( \angle 3, \angle 4, \) and \( \angle 5 \). Then find and record \( m \angle 2 \) and use it to calculate these same measures. Organize your results in two tables.
   c. VERBAL Explain how you used \( m \angle 1 \) to find the measures of \( \angle 3, \angle 4, \) and \( \angle 5 \). Then explain how you used \( m \angle 2 \) to find these same measures.
   d. ALGEBRAIC If \( m \angle 1 = x \), write an expression for the measures of \( \angle 3, \angle 4, \) and \( \angle 5 \). Likewise, if \( m \angle 2 = x \), write an an expression for these same angle measures.

45. CHALLENGE In the figure at the right, if \( \triangle WJZ \) is equilateral and \( \angle ZWP \equiv \angle WJM \equiv \angle JZL \), prove that \( \overline{WP} \equiv \overline{ZL} \equiv \overline{JM} \).

REASONING Determine whether the following statements are sometimes, always, or never true. Explain.

46. If the measure of the vertex angle of an isosceles triangle is an integer, then the measure of each base angle is an integer.
Name:

47. If the measures of the base angles of an isosceles triangle are integers, then the measure of its vertex angle is odd.

48. **FIND THE ERROR** Alexis and Miguela are finding \( m \angle G \) in the figure shown. Alexis says that \( m \angle G = 35 \), while Miguela says that \( m \angle G = 60 \). Is either of them correct? Explain your reasoning.

![Diagram of triangle GEF with angle F labeled 70°.]

49. **OPEN ENDED** If possible, draw an isosceles triangle with base angles that are obtuse. If it is not possible, explain why not.

50. **REASONING** In isosceles \( \triangle ABC \), \( m \angle B = 90 \). Draw the triangle. Indicate the congruent sides and label each angle with its measure.

51. **WRITING IN MATH** Explain how you can use the measure of a base angle of an isosceles triangle to find the measure of the vertex angle.

52. **ALGEBRA** What quantity should be added to both sides of this equation to complete the square?
\[ x^2 - 10x = 3 \]
A \(-25\)
B \(-5\)
C \(5\)
D \(25\)

53. **SHORT RESPONSE** In a school of 375 students, 150 students play sports and 70 students are involved in the community service club. 30 students play sports and are involved in the community service club. How many students are not involved in either sports or the community service club?

54. In the figure below, \( \overline{AE} \) and \( \overline{BD} \) bisect each other at point \( C \).

![Diagram of triangle ABC with points E, D, and C labeled.]

Which additional piece of information would be enough to prove that \( \overline{DE} \cong \overline{DC} \)?
F \( \angle A \cong \angle BCA \)
G \( \angle B \cong \angle D \)
H \( \angle ACB \cong \angle EDC \)
J \( \angle A \cong \angle B \)

55. **ACT/SAT** If \( x = -3 \), then \( 4x^2 - 7x + 5 = \)
A 2
B 20
C 42
D 62
56. If $m \angle ADC = 35$, $m \angle ABC = 35$, $m \angle DAC = 26$, and $m \angle BAC = 26$, determine whether $\triangle ADC \cong \triangle ABC$.

![Triangle Diagram]

Determine whether $\triangle STU \cong \triangle XYZ$. Explain.

57. $S(0, 5)$, $T(0, 0)$, $U(1, 1)$, $X(4, 8)$, $Y(4, 3)$, $Z(6, 3)$

58. $S(2, 2)$, $T(4, 6)$, $U(3, 1)$, $X(-2, -2)$, $Y(-4, 6)$, $Z(-3, 1)$

59. **PHOTOGRAPHY** Film is fed through a traditional camera by gears that catch the perforation in the film. The distance from $A$ to $C$ is the same as the distance from $B$ to $D$. Show that the two perforated strips are the same width.

![Perforated Strips Diagram]

State the property that justifies each statement.

60. If $x(y + z) = a$, then $xy + xz = a$.

61. If $n - 17 = 39$, then $n = 56$.

62. If $m \angle P + m \angle Q = 110$ and $m \angle R = 110$, then $m \angle P + m \angle Q = m \angle R$.

63. If $cv = md$ and $md = 15$, then $cv = 15$.

Refer to the figure at the right.

![Figures Diagram]

64. How many planes appear in this figure?

65. Name three points that are collinear.

66. Are points $A$, $C$, $D$, and $J$ coplanar?
67. **PROOF** If $\angle ACB \cong \angle ABC$, then $\angle XCA \cong \angle YBA$.

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**Explore 4-7 Graphing Technology Lab: Congruence Transformations**

- Analyze the Results

Determine whether $\triangle ABC$ and $\triangle A'B'C'$ are congruent. Explain your reasoning.

1. Activity 1

2. Activity 2

3. Activity 3

4. Explain why $\triangle A'B'C'$ in Activity 3 does not appear to be congruent to $\triangle ABC$.

5. **MAKE A CONJECTURE** Repeat Activities 1–3 using a different triangle $XYZ$. Analyze your results and compare them to those found in Exercises 1–3. Make a conjecture as to the relationship between a triangle and its transformed image under a translation, reflection, or a rotation.
4-7 Congruence Transformations - Check Your Understanding

Identify the type of congruence transformation shown as a reflection, translation, or rotation.

1.

2.

3.
4-7 Congruence Transformations - Practice and Problem Solving

Identify the type of congruence transformation shown as a reflection, translation, or rotation.

7.
Identify the type of congruence transformation shown in each picture as a reflection, translation, or rotation.
14. 

15. 

16. 

**COORDINATE GEOMETRY** Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation.

17. \( M(-7, -1), \ P(-7, -7), \ R(-1, -4); \ T(7, -1), \ V(7, -7), \ S(1, -4) \)

18. \( A(3, 9), \ B(3, 7), \ C(7, 7); \ S(3, 5), \ T(3, 3), \ R(7, 3) \)

19. \( A(-4, 5), \ B(0, 2), \ C(-4, 2); \ X(-5, -4), \ Y(-2, 0), \ Z(-2, -4) \)

20. \( A(2, 2), \ B(4, 7), \ C(6, 2); \ D(2, -2), \ F(4, -7), \ G(6, -2) \)

**CONSTRUCTION** Identify the type of congruence transformation performed on each given triangle to generate the other triangle in the truss shown below.

21. \( \triangle NML \) to \( \triangle CDE \)

22. \( \triangle EFD \) to \( \triangle GHF \)

23. \( \triangle CBJ \) to \( \triangle NQP \)
AMUSEMENT RIDES Identify the type of congruence transformation shown in each picture as a reflection, translation, or rotation.

24. 

25. 

26. 

27. SCHOOL Identify the transformations that are used to open a combination lock on a locker. If appropriate, identify the line of symmetry or center of rotation.

28. ALPHABET Determine which letters of the alphabet have vertical and/or horizontal lines of reflection.

29. DECORATING Tionne is redecorating her bedroom. She can use stencils or a stamp to create the design shown.
   a. If Tionne used the stencil, what type of transformation was used to produce each flower in the design?
   b. What type of transformation was used if she used the stamp to produce each flower in the design?
30. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the ordered pairs of a figure and its translated image.
   a. **GEOMETRIC** Draw congruent rectangles ABCD and WXYZ on a coordinate plane.
   b. **VERBAL** How do you get from a vertex on ABCD to the corresponding vertex on WXYZ using only horizontal and vertical movement?
   c. **TABULAR** Copy the table shown. Use your rectangles to fill in the x-coordinates, the y-coordinates, and the unknown value in the transformation column.
   d. **ALGEBRAIC** Complete the following notation that represents the rule for the translation \(ABCD \rightarrow WXYZ\): \((x, y) \rightarrow (x + 3, y + 3)\)

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Transformation</th>
<th>Rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCD</td>
<td>(x + 3, y + 3)</td>
<td>WXYZ</td>
</tr>
<tr>
<td>A(0, 0)</td>
<td></td>
<td>W(0, 0)</td>
</tr>
<tr>
<td>B(1, 0)</td>
<td>(1 + 3, 0 + 3)</td>
<td>X(4, 0)</td>
</tr>
<tr>
<td>C(1, 1)</td>
<td>(1 + 3, 1 + 3)</td>
<td>Y(4, 1)</td>
</tr>
<tr>
<td>D(0, 1)</td>
<td>(0 + 3, 1 + 3)</td>
<td>Z(3, 1)</td>
</tr>
</tbody>
</table>

31. **CHALLENGE** Use the diagram at the right.
   a. Identify two transformations of Triangle 1 that can result in Triangle 2.
   b. What must be true of the triangles in order for more than one transformation on a preimage to result in the same image? Explain your reasoning.

   ![Diagram of triangles]

32. **REASONING** A **dilation** is another type of transformation. In the diagram, a small paper clip has been dilated to produce a larger paper clip. Explain why dilations are not a congruence transformation.

33. reflection

34. translation

35. rotation

**OPEN ENDED** Describe a real-world example of each of the following transformations, other than those given in this lesson.
36. **WRITING IN MATH** In the diagram at the right \(\triangle DEF\) is called a *glide reflection* of \(\triangle ABC\). Based on the diagram, define a glide reflection. Is a glide reflection a congruence transformation? Explain your reasoning.

![Diagram of \(\triangle DEF\) and \(\triangle ABC\)](image)

37. **SHORT RESPONSE** Cindy is shopping for a new desk chair at a store where the desk chairs are 50% off. She also has a coupon for 50% off any one item. Cindy thinks that she can now get the desk chair for free. Is this true? If not, what will be the percent off she will receive with both the sale and the coupon?

38. Identify the congruence transformation shown.

- A dilation
- B reflection
- C rotation
- D translation

39. Look at the graph below. What is the slope of the line shown?

![Graph with points F, G, H, J marked on the x-axis]

- F: -2
- G: -1
- H: 1
- J: 2

40. **ACT/SAT** What is the y-intercept of the line determined by the equation \(3x - 4 = 12y - 3\)?

   - A -12
   - B \(-\frac{1}{12}\)
   - C \(\frac{1}{12}\)
   - D 12

  Find each measure.
Name:

41. $YZ$

42. $m\angle JLK$

43. $AB$

PROOF Write a paragraph proof.

44. Given: $\angle YWZ \equiv \angle XZW$ and $\angle YZW \equiv \angle XWZ$
Prove: $\triangle WXZ \equiv \triangle ZYW$

45. **ROLLER COASTERS** The sign in front of the Electric Storm roller coaster states that all riders must be at least 54 inches tall to ride. If Andy is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion?

Find the coordinates of the midpoint of a segment with the given endpoints.

46. $A(10, -12), C(5, -6)$

47. $A(13, 14), C(3, 5)$

48. $A(-28, 8), C(-10, 2)$

49. $A(-12, 2), C(-3, 5)$

50. $A(0, 0), C(3, -4)$

51. $A(2, 14), C(0, 5)$
4-8 Triangles and Coordinate Proof - Check Your Understanding

1. Position and label each triangle on the coordinate plane.
   
   right $\triangle ABC$ with legs $\overline{AC}$ and $\overline{AB}$ so that $\overline{AC}$ is $2a$ units long and leg $\overline{AB}$ is $2b$ units long

2. isosceles $\triangle FGH$ with base $\overline{FG}$ that is $2a$ units long

   Name the missing coordinate(s) of each triangle.

3. 

4. 

5. Write a coordinate proof to show that $\triangle FGH \cong \triangle FDC$.

6. **FLAGS** Write a coordinate proof to prove that the large triangle in the center of the flag is isosceles. The dimensions of the flag are 4 feet by 6 feet and point $B$ of the triangle bisects the bottom of the flag.
4-8 Triangles and Coordinate Proof - Practice and Problem Solving

Position and label each triangle on the coordinate plane.
7. isosceles \( \triangle ABC \) with base \( \overline{AB} \) that is \( a \) units long

8. right \( \triangle YZ \) with hypotenuse \( YZ \), the length of \( \overline{XY} \) is \( b \) units long, and the length of \( \overline{XZ} \) is three times the length of \( \overline{XY} \)

9. isosceles right \( \triangle RST \) with hypotenuse \( RS \) and legs \( 3a \) units long

10. right \( \triangle JKL \) with legs \( JK \) and \( KL \) so that \( JK \) is \( a \) units long and leg \( KL \) is \( 4b \) units long

11. equilateral \( \triangle GHJ \) with sides \( \frac{1}{2}a \) units long

12. equilateral \( \triangle DEF \) with sides \( 4b \) units long

Name the missing coordinate(s) of each triangle.
Name:

17. \[ \text{PROOF Write a coordinate proof for each statement.} \]

19. The segments joining the base vertices to the midpoints of the legs of an isosceles triangle are congruent.

20. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.

\[ \text{PROOF Write a coordinate proof for each statement.} \]

21. The measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

22. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one half the length of the third side.

23. \textbf{RESEARCH TRIANGLE} The cities of Raleigh, Durham, and Chapel Hill, North Carolina, form what is known as the Research Triangle. The approximate latitude and longitude of Raleigh are 35.82°N 29.9°W, of Durham are 35.97°N 78.9°W, and of Chapel Hill are 35.92°N 79.03°W. Show that the triangle formed by these three cities is scalene.

24. \textbf{PARTY PLANNING} Three friends live in houses with backyards adjacent to a neighborhood bike path. They decide to have a round-robin party using their three homes, inviting their friends to start at one house and then move to each of the other two. If one friend’s house is centered at the origin, then the location of the other homes are (5, 12) and (13, 0). Write a coordinate proof to prove that the triangle formed by these three homes is isosceles.

\[ \text{Draw } \triangle XYZ \text{ and find the slope of each side of the triangle. Determine whether the triangle is a right triangle. Explain} \]

25. \[ X(0, 0), Y(2h, 2h), Z(4h, 0) \]

26. \[ X(0, 0), Y(1, h), Z(2h, 0) \]

27. \textbf{CAMPING} Two families set up tents at a state park. If the ranger’s station is located at (0, 0), and the locations of the tents are (0, 25) and (12, 9), write a coordinate proof to prove that the figure formed by the locations of the ranger’s station and the two tents is a right triangle.
28. PROOF Write a coordinate proof to prove that \( \triangle ABC \) is an isosceles triangle if the vertices are \( A(0, 0) \), \( B(a, b) \), and \( C(2a, 0) \).

29. WATER SPORTS Three personal watercraft vehicles launch from the same dock. The first vehicle leaves the dock traveling due northeast, while the second vehicle travels due northwest. Meanwhile, the third vehicle leaves the dock traveling due north.

The first and second vehicles stop about 300 yards from the dock, while the third stops about 212 yards from the dock.

a. If the dock is located at \((0, 0)\), sketch a graph to represent this situation. What is the equation of the line along which the first vehicle lies? What is the equation of the line along which the second vehicle lies? Explain your reasoning.

b. Write a coordinate proof to prove that the dock, the first vehicle, and the second vehicle form an isosceles right triangle.

c. Find the coordinates of the locations of all three watercrafts. Explain your reasoning.

d. Write a coordinate proof to prove that the positions of all three watercrafts are approximately collinear and that the third watercraft is at the midpoint between the other two.

30. REASONING The midpoints of the sides of a triangle are located at \((a, 0)\), \((2a, b)\) and \((a, b)\). If one vertex is located at the origin, what are the coordinates of the other vertices? Explain your reasoning.

CHALLENGE Find the coordinates of point \(L\) so \( \triangle JKL \) is the indicated type of triangle. Point \(J\) has coordinates \((0, 0)\) and point \(K\) has coordinates \((2a, 2b)\).

31. scalene triangle

32. right triangle

33. isosceles triangle

34. OPEN ENDED Draw an isosceles right triangle on the coordinate plane so that the midpoint of its hypotenuse is the origin. Label the coordinates of each vertex.

35. CHALLENGE Use a coordinate proof to show that if you add \(n\) units to each \(x\)-coordinate of the vertices of a triangle and \(m\) to each \(y\)-coordinate, the resulting figure is congruent to the original triangle.

36. REASONING A triangle has vertex coordinates \((0, 0)\) and \((a, 0)\). If the coordinates of the third vertex are in terms of \(a\), and the triangle is isosceles, identify the coordinates and position the triangle on the coordinate plane.

37. WRITING IN MATH Explain why following each guideline below for placing a triangle on the coordinate plane is helpful in proving coordinate proofs.

a. Use the origin as a vertex of the triangle.

b. Place at least one side of the triangle on the \(x\)- or \(y\)-axis.

c. Keep the triangle within the first quadrant if possible.
38. **GRIDDED RESPONSE** In the figure below, $m \angle B = 76$. The measure of $\angle A$ is half the measure of $\angle B$. What is $m \angle C$?

![Diagram of triangle](image1)

39. **ALGEBRA** What is the $x$-coordinate of the solution to the system of equations shown below?

$$\begin{align*}
2x - 3y &= 3 \\
-4x + 2y &= -18
\end{align*}$$

A $-6$
B $-3$
C $3$
D $6$

40. What are the coordinates of point $R$ in the triangle?

![Triangle with points](image2)

- F $\left(\frac{a}{2}, a\right)$
- G $(a, b)$
- H $\left(\frac{b}{2}, a\right)$
- J $\left(\frac{b}{2}, \frac{a}{2}\right)$

41. **SAT/ACT** For all $x$, $17x^5 + 3x^2 + 2 - (-4x^5 + 3x^3 - 2) =

A $13x^5 + 3x^3 + 3x^2$
B $21x^5 - 3x^3 + 3x^2 + 4$
C $13x^5 + 6x^2 + 4$
D $21x^5 + 3x^2 + 3x^3$

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

42. ![Grid with shapes](image3)
43. Refer to the figure at the right.

44. Name two congruent angles.

45. Name two congruent segments.

46. Name a pair of congruent triangles.

48. **RAMPs** The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch.
   a. Determine the slope represented by this requirement.
   b. The maximum length that the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

Find the distance between each pair of points. Round to the nearest tenth.

49. $X(5, 4)$ and $Y(2, 1)$

50. $A(1, 5)$ and $B(-2, -3)$

51. $J(-2, 6)$ and $K(1, 4)$
Study Guide and Review - Vocabulary Check - Chapter 4

State whether each sentence is true or false. If false, replace the underlined word or phrase to make a true sentence.

1. An equiangular triangle is also an example of an acute triangle.

2. A triangle with an angle that measures greater than $90^\circ$ is a right triangle.

3. An equilateral triangle is always equiangular.

4. A scalene triangle has at least two congruent sides.

5. The vertex angles of an isosceles triangle are congruent.

6. An included side is the side located between two consecutive angles of a polygon.

7. The three types of congruence transformations are rotation, reflection, and translation.

8. A rotation moves all points of a figure the same distance and in the same direction.

9. A flow proof uses figures in the coordinate plane and algebra to prove geometric concepts.

10. The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.
Study Guide and Review - Lesson-by-Lesson Review - Chapter 4

11. \( \triangle ADB \)

12. \( \triangle BCD \)

13. \( \triangle ABC \)

ALGEBRA Find \( x \) and the measures of the unknown sides of each triangle.
16. **MAPS** The distance from Chicago to Cleveland to Cincinnati and back to Chicago is 900 miles. The distance from Chicago to Cleveland is 50 miles more than the distance from Cincinnati to Chicago, and the distance from Cleveland to Cincinnati is 50 miles less than the distance from Cincinnati to Chicago. Find each distance and classify the triangle formed by the three cities.

Find the measure of each numbered angle.

17. $\angle 1$

18. $\angle 2$

19. $\angle 3$

20. **HOUSES** The roof support on Lamar's house is in the shape of an isosceles triangle with base angles of 38°. Find $x$.

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

21.
22. [Diagram of triangles labeled X, Y, Z, L, K with angles and side lengths]

23. **MOSAIC TILING** A section of a mosaic tiling is shown. Name the triangles that appear to be congruent.

24. \(A(5, 2), B(1, 5), C(0, 0), X(-3, 3), Y(-7, 6), Z(-8, 1)\)

25. \(A(3, -1), B(3, 7), C(7, 7), X(-7, 0), Y(-7, 4), Z(1, 4)\)

   Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

26. [Diagram of a triangle with a line segment drawn through it]

27. [Diagram of a rectangle with a diagonal drawn]

28. **PARKS** The diagram shows a park in the shape of a pentagon with five sidewalks of equal length leading to a central point. If all the angles at the central point have the same measure, how could you prove that \(\triangle ABX \cong \triangle DCX\) ?

   Write a two-column proof.
29. Given: $AB \parallel DC, AB \cong DC$
Prove: $\triangle ABE \cong \triangle CDE$

30. **KITES** Denise’s kite is shown in the figure below. Given that $WY$ bisects both $\angle WYZ$ and $\angle XYZ$, prove that $\triangle WXY \cong \triangle WZY$.

![Kite diagram](image)

Find the value of each variable.

31.

![Triangle diagram](image)

32.

33. **PAINTING** Pam is painting using a wooden easel. The support bar on the easel forms an isosceles triangle with the two front supports. According to the figure below, what are the measures of the base angles of the triangle?

![Easel diagram](image)

Identify the type of congruence transformation shown as a **reflection**, **translation**, or **rotation**.
38. Triangle $ABC$ with vertices $A(1, 1)$, $B(2, 3)$, and $C(3, -1)$ is a transformation of $\triangle MNO$ with vertices $M(-1, 1)$, $N(-2, 3)$, and $O(-3, -1)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Position and label each triangle on the coordinate plane.

39. right $\triangle MNO$ with right angle at point $M$ and legs of lengths $a$ and $2a$.

40. isosceles $\triangle WXY$ with height $h$ and base $\overline{WY}$ with length $2a$.

41. GEOGRAPHY Jorge plotted the cities of Dallas, San Antonio, and Houston as shown. Write a coordinate proof to show that the triangle formed by these cities is scalene.
Chapter 4 - Congruent Triangles - Practice Test - Chapter 4

Classify each triangle as acute, equiangular, obtuse, or right.

1. $\triangle ABD$
2. $\triangle ABC$
3. $\triangle BDC$

Find the measure of each numbered angle.

4. $\angle 1$
5. $\angle 2$
6. $\angle 3$
7. $\angle 4$

In the diagram, $\triangle RST \cong \triangle XYZ$.

8. Find $x$.
10. **PROOF** Write a flow proof.
   Given: $\overline{XY} \parallel \overline{WZ}$ and $\overline{XW} \parallel \overline{YZ}$
   Prove: $\triangle XYZ \cong \triangle YZX$

11. **MULTIPLE CHOICE** Find $x$.

   ![Diagram with angles 116° and 72°]

   - A 36
   - B 32
   - C 28
   - D 22

12. Determine whether $\triangle TJD \cong \triangle SEK$ given $T(-4, -2)$, $J(0, 5)$, $D(1, -1)$, $S(-1, 3)$, $E(3, 10)$, and $K(4, 4)$. Explain.

   Determine which postulate or theorem can be used to prove each pair of triangles congruent. If it is not possible to prove them congruent, write *not possible*.

13. 

14. 

15. 

16. 

17. **LANDSCAPING** Angie has laid out a design for a garden consisting of two triangular areas as shown below. The points are \( A(0, 0) \), \( B(0, 5) \), \( C(3, 5) \), \( D(6, 5) \), and \( E(6, 0) \). Name the type of congruence transformation for the preimage \( \triangle ABC \) to \( \triangle EDC \).

![Diagram of the garden design showing points A, B, C, D, and E.]

Find the measure of each numbered angle.

18. \( \angle 1 \)

19. \( \angle 2 \)

20. **PROOF** \( \triangle ABC \) is a right isosceles triangle with hypotenuse \( \overline{AB} \). \( M \) is the midpoint of \( \overline{AB} \). Write a coordinate proof to show that \( \overline{CM} \) is perpendicular to \( \overline{AB} \).
Chapter 4 - Congruent Triangles - Preparing for Standardized Tests - Chapter 4

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. Classify \( \triangle DEF \) according to its angle measures.

2. In the figure below, \( \triangle RST \cong \triangle VUT \). What is the area of \( \triangle RST \)?

3. A farmer needs to make a 1000-square-foot rectangular enclosure for her cattle. She wants to save money by purchasing the least amount of fencing possible to enclose the area. What whole-number dimensions will require the least amount of fencing?

4. What is the \( m \angle 1 \) in degrees?

5. Write an equation of the line containing the points (2, 4) and (0, -2)
Chapter 4 - Congruent Triangles - Standardized Test Practice
- Cumulative, Chapters 1-4

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.
1. If $m \angle 1 = 110^\circ$, what must $m \angle 2$ equal for lines $x$ and $z$ to be parallel?

A $30^\circ$  
B $60^\circ$  
C $70^\circ$  
D $110^\circ$

2. Which of the following terms best describes the transformation below?

F dilation  
G reflection  
H rotation  
J translation

3. Classify the triangle below according to its side lengths.

A equilateral  
B isosceles  
C right  
D scalene

4. Given: $\overline{WX} \cong \overline{JK}$, $\overline{XY} \cong \overline{IK}$, $\angle X \cong \angle K$

Which of the following lists the correct triangle congruence?

F $\triangle WXY \cong \triangle KIJ$  
G $\triangle WXY \cong \triangle IJK$  
H $\triangle WXY \cong \triangle JKI$  
J $\triangle WXY \cong \triangle IJK$
5. Suppose the dimensions of the prism below are tripled. By what factor will the volume of the prism increase?

- A 3
- B 9
- C 27
- D 81

6. What is the measure of angle $R$ below?

- F $57^\circ$
- G $59^\circ$
- H $65^\circ$
- J $68^\circ$

7. Suppose one base angle of an isosceles triangle has a measure of $44^\circ$. What is the measure of the vertex angle?

- A $108^\circ$
- B $92^\circ$
- C $56^\circ$
- D $44^\circ$

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**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

8. **GRIDDED RESPONSE** In the figure below, $\triangle NDG \cong \triangle LGD$. What is the value of $x$?

9. **GRIDDED RESPONSE** Suppose line $\ell$ contains points $A$, $B$, and $C$. If $AB = 7$ inches, $AC = 32$ inches, and point $B$ is between points $A$ and $C$, what is the length of $BC$? Express your answer in inches.

10. Write the converse of the statement.

   *If you are the winner, then I am the loser.*

11. Use the figure and the given information below.

   ![Diagram](image)

   **Given:** $JT \perp AP$

   $\angle 1 \equiv \angle 2$

   Which congruence theorem could you use to prove $\triangle PTJ \cong \triangle ATJ$ with only the information given? Explain.
12. Write an equation in slope intercept form for the line which goes through the points (0, 3) and (4, -5).

13. **GRIDDED RESPONSE** Find $m \angle TUV$ in the figure.

14. Suppose two sides of triangle $ABC$ are congruent to two sides of triangle $MNO$. Also, suppose one of the nonincluded angles of $\Delta ABC$ is congruent to one of the nonincluded angles of $\Delta MNO$. Are the triangles congruent? If so, write a paragraph proof showing the congruence. If not, sketch a counterexample.

*Record your answers on a sheet of paper. Show your work.*

15. Use a coordinate grid to write a coordinate proof of the following statement.

   *If the vertices of a triangle are $A(0, 0)$, $B(2a, b)$, and $C(4a, 0)$, then the triangle is isosceles.*

   a. Plot the vertices on a coordinate grid to model the problem.
   b. Use the Distance Formula to write an expression for $AB$.
   c. Use the Distance Formula to write an expression for $BC$.
   d. Use your results from parts $b$ and $c$ to draw a conclusion about